



Two planes on merging routes are:  
 --the same distance from the intersection  
 --traveling at the same speed.

## SMART SKIES™ FLYBY MATH™

### Distance-Rate-Time Problems in Air Traffic Control for Grades 5 - 9

## AIR TRAFFIC CONTROL PROBLEM 1

### Teacher Guide with Answer Sheets

#### Overview of Air Traffic Control Problem 1

In this Air Traffic Control (ATC) Problem, students will determine if two airplanes traveling on different merging routes will conflict with (meet) one another at the intersection of their flight routes.

The airplanes are each the **same distance from the point of intersection**.  
 The airplanes are traveling at the **same constant (fixed) speeds**.

Of the five *FlyBy Math™* ATC Problems, this scenario is the simplest two-plane scenario.

#### Objectives

Students will determine the following:

- If two planes are traveling at the same constant (fixed) speed on two different routes and the planes are the same distance from the point where the two routes come together, the planes will arrive at the intersection at the same time. So the planes will meet at the point where the routes come together.

#### Materials

Student handouts:

ATC Problem 1 Student Workbook  
 ATC Problem 1 Assessment Package (optional)

The student handouts are available on the *FlyBy Math™* website:

<http://www.smartskies.nasa.gov/flyby>



### Introducing Your Students to the ATC Problem

*You may want to show the FlyBy Math™ video clips to introduce your students to the air traffic control system.*

*(For more detail, see the FlyBy Math™ Educator Guide.)*

### Student Workbook

Materials for the experiment:

- sidewalk chalk or masking tape or cashier's tape or a knotted rope
- measuring tape or ruler
- marking pens (optional)
- 1 stopwatch or 1 watch with a sweep second hand or 1 digital watch that indicates seconds
- pencils
- signs (available on the *FlyBy Math™* website) identifying pilots, controllers, and NASA scientists
- clipboard (optional)

To help your students understand the problem, you can ask them to consider this related problem that is set in a more familiar context:

Two students, Ana and Alex, plan to meet at the movies. Each student lives the same distance from the theater. Ana and Alex will each leave their homes at the same time and walk at the same constant (fixed) speed.

You can ask your students these questions:

Will Ana and Alex arrive at the movie theater at the same time? Why or why not?

In particular, if your students think Ana and Alex will arrive at different times, ask them who will arrive first and why.

Since the students are walking at the same constant (fixed) speed and each must travel the same distance to the theater, students may realize that the students will arrive at the theater at the same time.

For a detailed description of the Student Workbook features found in each ATC Problem, see the *FlyBy Math™* Educator Guide.

The following section addresses *special features* of the ATC Problem 1 Workbook.

#### Read the Problem

The speed of each airplane is 1/2 foot/second. Each airplane is 20 feet from the point of intersection.

Note: The speeds and distances were chosen to reflect the classroom experiment that the students will conduct and are not related to real-world parameters.



*As a problem enhancement, you may want to ask your students to solve the problem using real-world data.*

In a real-world scenario, each plane might be traveling at 400 nautical miles per hour. Each plane might be 40 nautical miles from the point of intersection.

An international nautical mile is 1,852 meters.

A nautical mile per hour is called a “knot”.

### **Set Up and Do the Experiment**

A complete description of this section is contained in the *FlyBy Math™* Educator Guide.

### **Do the Calculations**

Each of the six calculation methods is described in the *FlyBy Math™* Educator Guide.

One method, Graph Two Linear Equations, is described in greater detail below.

#### **--Graph Two Linear Equations**

Caution: Students may confuse the path of a plane with the graph of the plane’s distance traveled as a function of time. In particular, students need to understand that the planes meet only at the intersection, but the graphs intersect at each point.

### **Analyze Your Results**

As part of the Analysis, you may also want to ask your students to create a similar problem in a different setting. They have already considered a problem in which two students walk from their respective homes to a movie theater. (See the *Introducing Your Students to the ATC Problem* section of this document.)

Now, you might suggest they consider two cars traveling in parallel lanes on the same road, with the two lanes merging into one lane. Each car is traveling at the same constant (fixed) speed. The cars are each the same distance from the merge. Students should realize that the cars will arrive at the merge at the same time.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would *not* be appropriate because a launched rocket typically accelerates and therefore its speed is not constant.)

### **Answers and Explanations**

The first part of this section summarizes the answers to the key questions posed in the ATC Problem. The remainder of this section is organized by activity and includes graphs, diagrams, and answers to individual questions, as well as discussions of the general problems posed in the analysis activity and the posttest.

**Answer Summary**

The speed of Flight WAL27 is  $1/2$  foot/second, so the plane travels  $1/2$  foot in 1 second.

The speed of Flight NAL63 is  $1/2$  foot/second, so the plane travels  $1/2$  foot in 1 second.

Flight WAL27 is 20 feet from the point of intersection.

Flight NAL63 is 20 feet from the point of intersection.

Since the planes are traveling at the same constant (fixed) speed and must travel the same distance to the point of intersection, a conflict will occur at the intersection.

In particular:

- It will take 40 seconds for Flight WAL27 to travel 20 feet to the point where the routes come together.
- It will take 40 seconds for Flight NAL63 to travel 20 feet to the point where the routes come together.
- Since the times are the same for each airplane, they will arrive at the same time and a conflict will occur.

**Answers by Activity**

Answers are provided for all worksheets including the Pretest, Student Workbook, and Posttest. Please see the following pages.

Note: Answers are given only for the numbered activity steps that require students to provide a numerical or verbal response. For example, if a step requires a student to trace or circle a portion of a diagram, that step is *not* included in the Answers.



## ANSWERS BY ACTIVITY

### Pretest

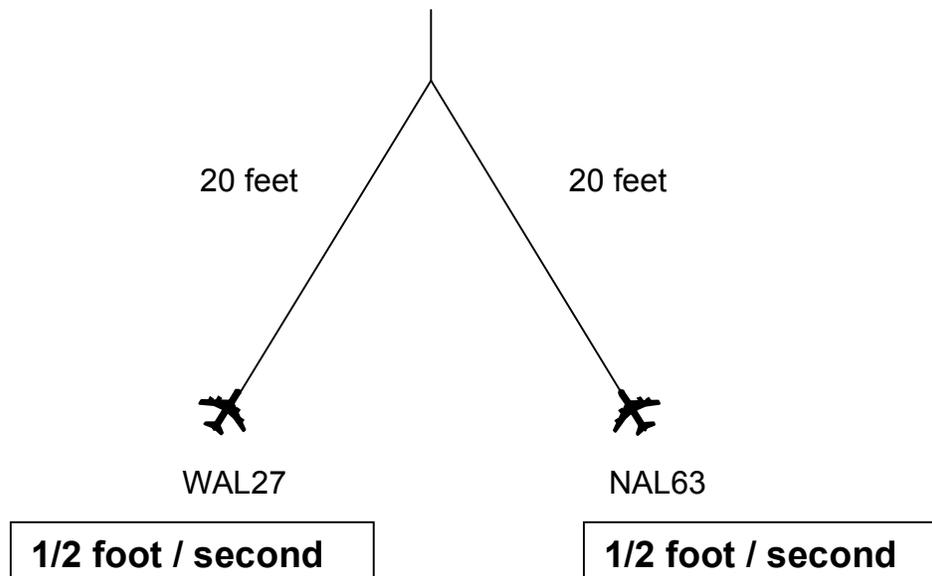
1. Do you think the planes will meet at the point where the two routes intersect?

**Yes**

Why or why not? **The planes are traveling at the same constant speed and each plane must travel the same distance to the point where the routes intersect. So the planes will meet at the intersection.**

2. If not, how many feet apart do you think the planes will be when the first plane reaches the point where the routes intersect?

**0 feet**

**Student Workbook: Read the Problem**

2. How far does WAL27 travel in 1 second? **1/2 foot**
3. How far does WAL27 travel in 10 seconds? **5 feet**
5. How far does NAL63 travel in 1 second? **1/2 foot**
6. How far does NAL63 travel in 10 seconds? **5 feet**

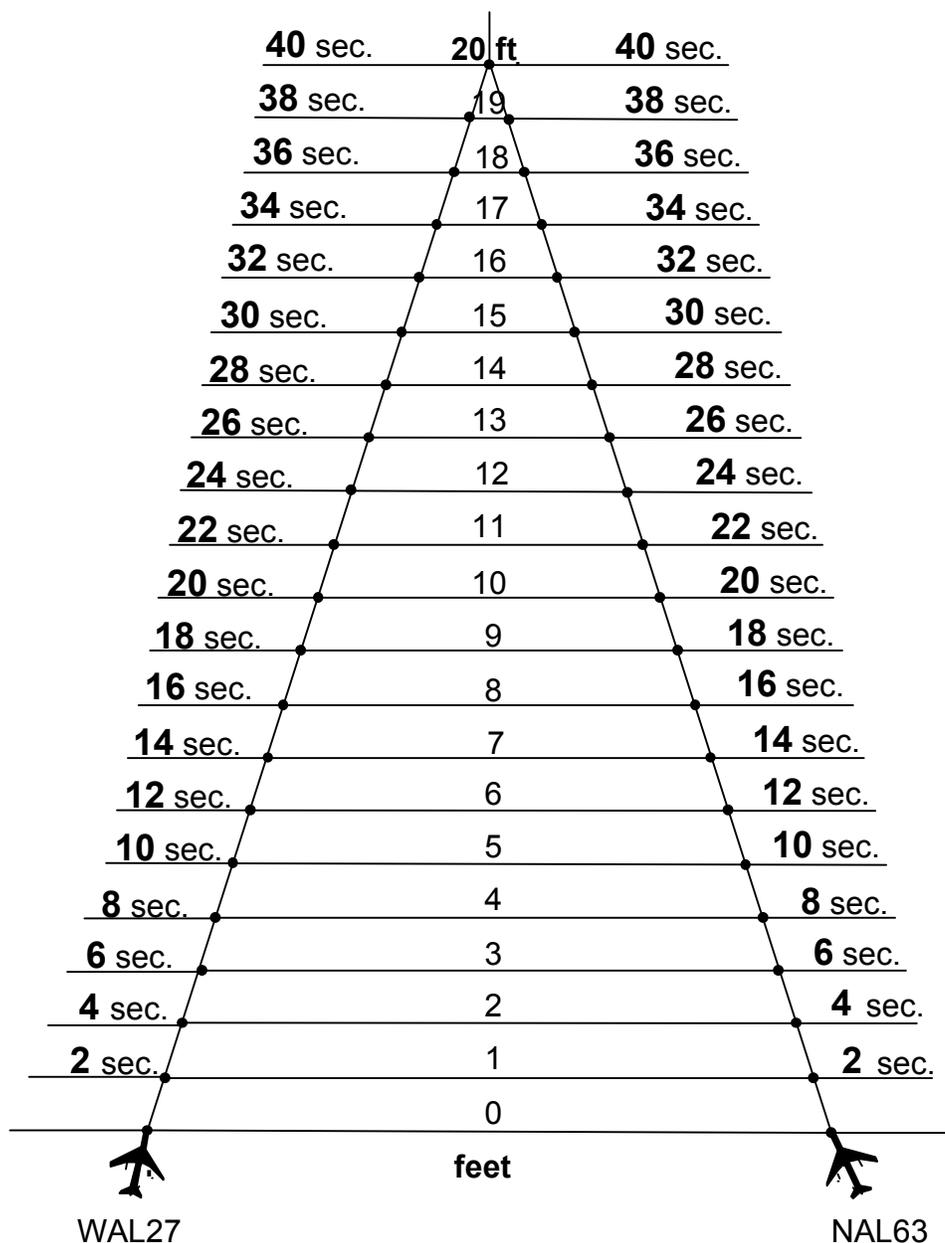

**Student Workbook: Do the Calculations—Count Feet and Seconds**

*Discussion: Each plane travels 1 foot in 2 seconds. Count by 2s to fill in the seconds in the blanks along the jet route.*

*Flight WAL27 will travel 20 feet in 40 seconds. (Students can also multiply 2 seconds per foot by 20 feet to obtain 40 seconds.)*

*Flight NAL63 will travel 20 feet in 40 seconds. (Students can also multiply 2 seconds per foot by 20 feet to obtain 40 seconds.)*

*Since the times are the same for each airplane, they will arrive at the same time and a conflict will occur.*



**Student Workbook: Do the Calculations—Count Feet and Seconds (cont.)**

7. How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27 40 seconds

NAL63 40 seconds

8. Did the planes meet at the point where  
the two routes intersect?

Yes

9. Why or why not?

**The planes each take 40 seconds to reach the intersection.  
So they arrive at the same time.**

10. If you think two planes will meet, what would you tell the air traffic controller  
to do to avoid a collision?

**Change the speed or change the route of one of the planes.**

13. You moved along each jet route, one foot at a time, to find the number of seconds it took each  
plane to travel to the point where the routes meet. Can you think of a faster way to find the number  
of seconds? If so, describe the faster way.

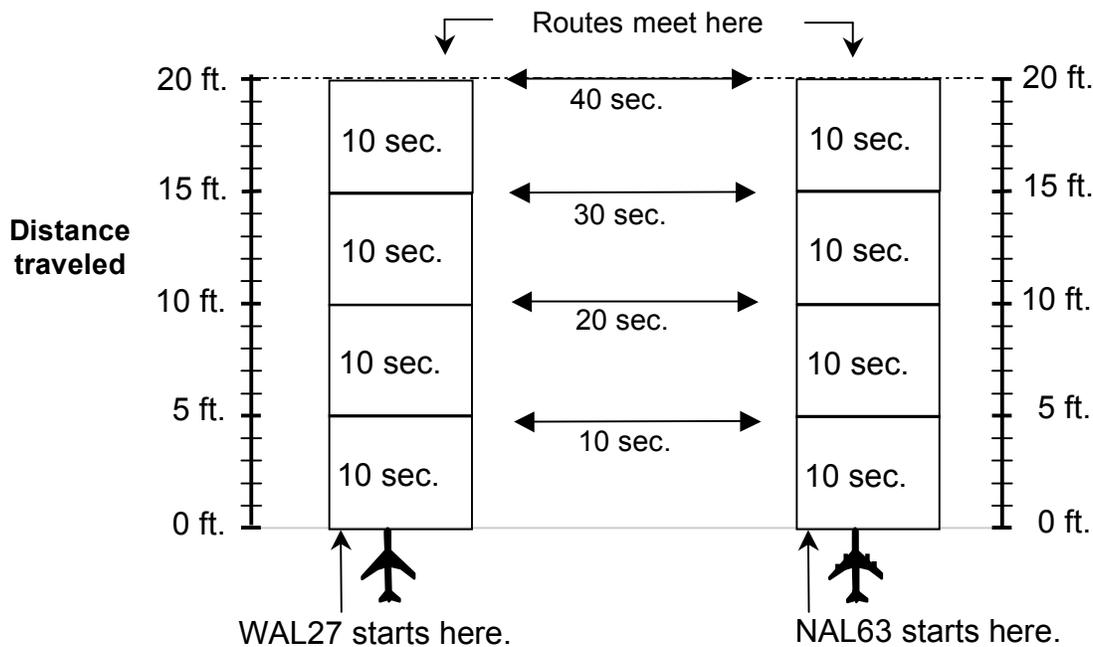
**For Flight WAL27, multiply 2 seconds per foot by 20 feet to obtain 40  
seconds. For Flight NAL63, multiply 2 seconds per foot by 20 feet to  
obtain 40 seconds.**



**Student Workbook: Do the Calculations—Draw Blocks**

*Discussion: Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following diagram shows the 5-foot blocks added together. To get the answer in seconds, add the 10-second blocks.*

*Since the times of arrival at the intersection are the same (40 seconds) for each plane, they will arrive at the same time and a conflict will occur.*



1. Each plane takes 10 seconds to travel ...

**5 feet**

15. How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27 40 seconds

NAL63 40 seconds



**Student Workbook: Do the Calculations—Draw Blocks (cont.)**

16. Did the planes meet at the point where  
the two routes intersect?

**Yes**

17. Why or why not?

**The planes each take 40 seconds to reach the intersection.  
So they arrive at the same time.**

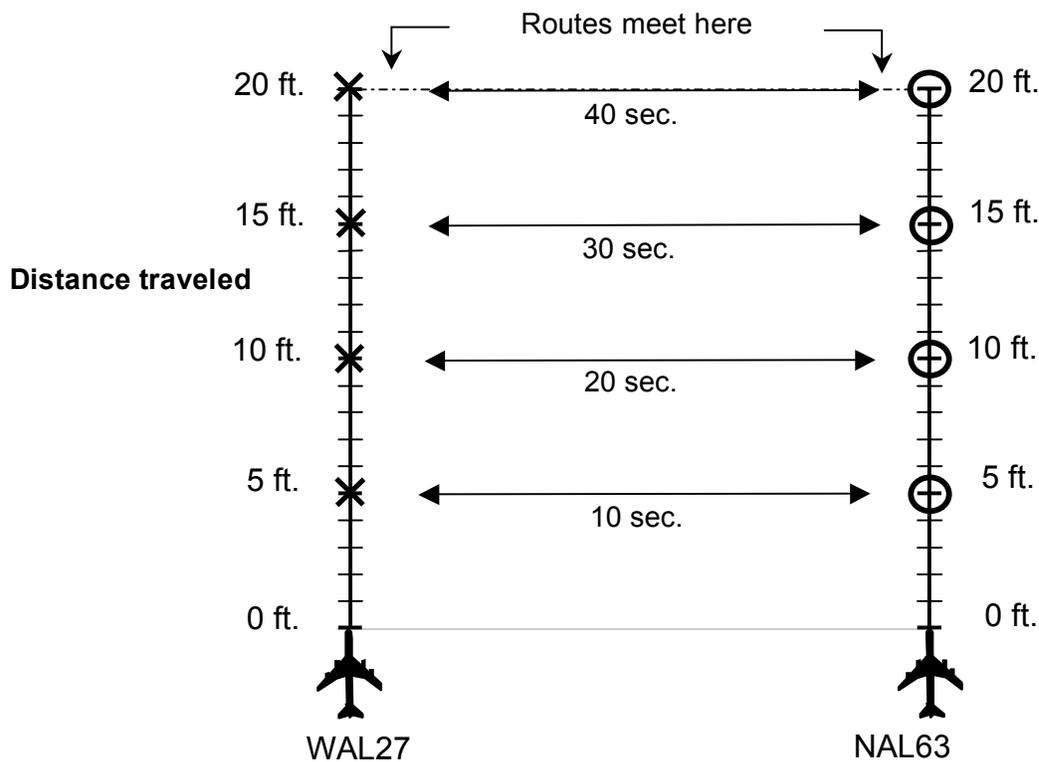
18. If you think two planes will meet, what would you tell the air traffic controller  
to do to avoid a collision?

**Change the speed or change the route of one of the planes.**


**Student Workbook: Do the Calculations—Plot Points on Lines**

*Discussion: Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following diagram shows the position of each plane at 10-second intervals.*

*For each plane, the time of arrival at the intersection is 40 seconds. So the planes arrive at the same time and a conflict will occur.*



14. How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27 40 seconds

NAL63 40 seconds

15. Did the planes meet at the point where  
the two routes intersect?

Yes



**Student Workbook: Do the Calculations—Plot Points on Lines (cont.)**

16. Why or why not?

**The planes each take 40 seconds to reach the intersection.**  
**So they arrive at the same time.**

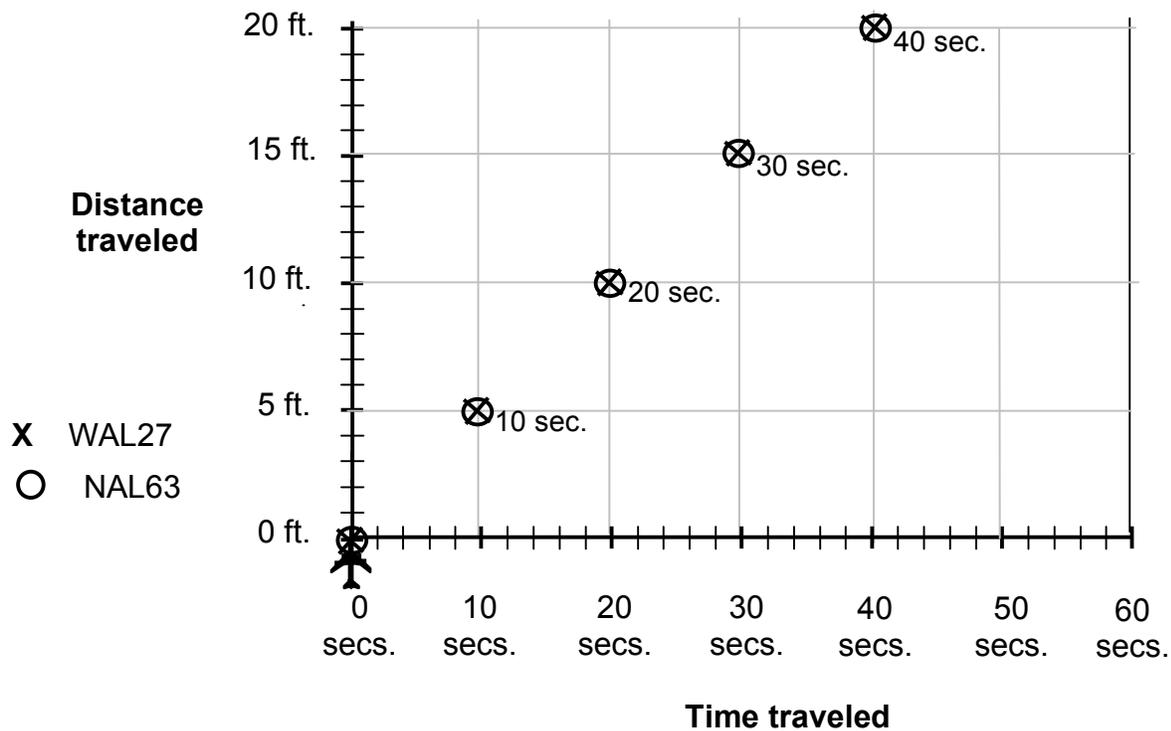
17. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

**Change the speed or change the route of one of the planes.**


**Student Workbook: Do the Calculations—Plot Points on a Grid**

*Discussion: Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following diagram shows the position of each plane at 10-second intervals.*

*For each plane, the time of arrival at the intersection is 40 seconds. So the planes arrive at the same time and a conflict will occur.*



13. How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27 40 seconds

NAL63 40 seconds

14. Did the planes meet at the point where the two routes intersect?

Yes



**Student Workbook: Do the Calculations—Plot Points on a Grid (cont.)**

15. Why or why not?

**The planes each take 40 seconds to reach the intersection.**  
**So they arrive at the same time.**

16. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

**Change the speed or change the route of one of the planes.**


**Student Workbook: Do the Calculations—Use a Formula**

*Discussion: Flight WAL27 arrives at the intersection in 40 seconds.*

*Flight NAL63 arrives at the intersection in 40 seconds.*

*Since the times are the same, a conflict will occur.*

1. -- In 4 seconds, each plane travels **0.5 feet/second** × **4 seconds** = **2.0** feet.

-- In 5 seconds, each plane travels **0.5 feet/second** × **5 seconds** = **2.5** feet.

-- In 6 seconds, each plane travels **0.5 feet/second** × **6 seconds** = **3.0** feet.

2. How could you use multiplication to find the distance each plane travels in 14 seconds? **Multiply 0.5 feet/second by 14 seconds.**

3. Use the formula

$$d = r \cdot t$$

to answer this question.

How many feet does each plane travel in 20 seconds? **10 feet**

4. Use the formula

$$t = \frac{d}{r}$$

to find the number of seconds for WAL27 to travel 20 feet to the point where the routes meet. (Hint: Divide 20 by 0.5).

$$t = \frac{20 \text{ feet}}{0.5 \text{ feet per second}} = \underline{\mathbf{40}} \text{ seconds}$$

**Student Workbook: Do the Calculations—Use a Formula (cont.)**

5. Use the same formula to find the number of seconds for NAL63 to travel 20 feet to the point where the routes meet.

$$t = \frac{20 \text{ feet}}{0.5 \text{ feet per second}} = \underline{40} \text{ seconds}$$

6. Did the planes meet at the point where the two routes intersect?

Yes

7. Why or why not?

**The planes each take 40 seconds to reach the intersection.**  
**So they arrive at the same time.**

8. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

**Change the speed or change the route of one of the planes.**


**Student Workbook: Do the Calculations—Graph Linear Equations**

*Discussion: Flight WAL27 arrives at the intersection in 40 seconds.*

*Flight NAL63 arrives at the intersection in 40 seconds.*

*Since the times are the same, a conflict will occur.*

1. Fill in the table for **WAL27**.

2. Fill in the table for **NAL63**.

$$d = 0.5 t$$

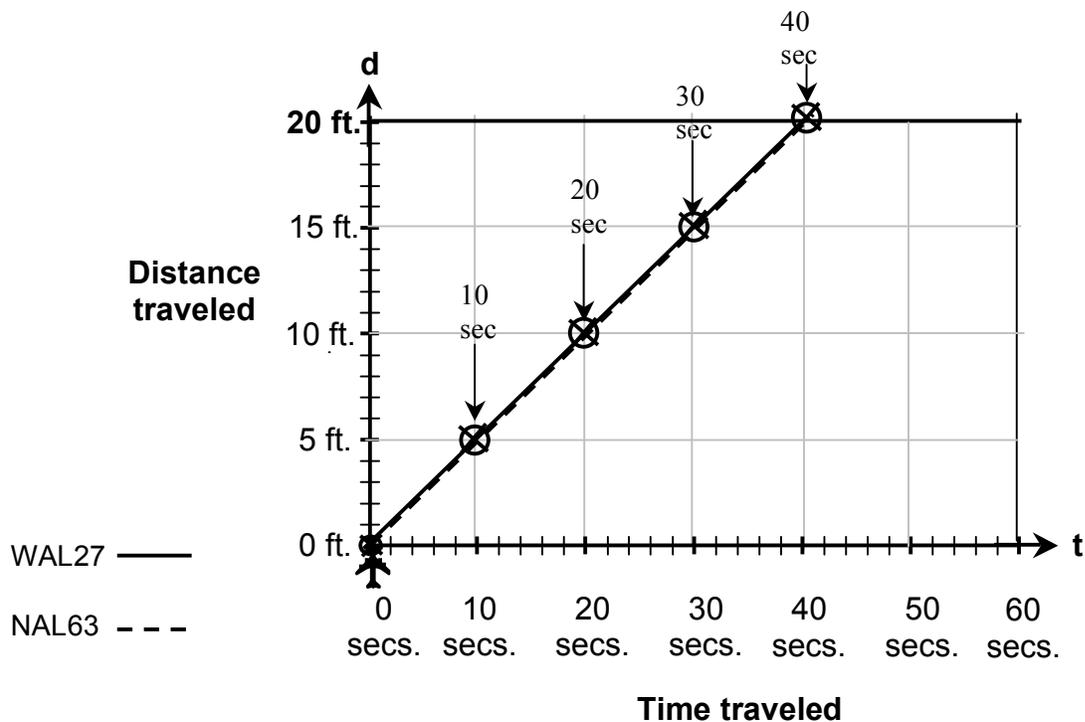
t seconds	d feet
0	<b>0</b>
10	<b>5</b>
20	<b>10</b>
30	<b>15</b>
40	<b>20</b>

$$d = 0.5 t$$

t seconds	d feet
0	<b>0</b>
10	<b>5</b>
20	<b>10</b>
30	<b>15</b>
40	<b>20</b>

3. Use an **X** to graph each point in the WAL27 table. Use a solid line to connect the points.

4. Use an **O** to graph each point in the NAL63 table. Use a dotted line to connect the points.





**Student Workbook: Do the Calculations—Graph Linear Equations (cont.)**

5. How many seconds will it take each plane to arrive at the point where the routes intersect?

WAL27 40 seconds

NAL63 40 seconds

6. Will the planes meet at the point where  
the two routes intersect?

Yes

7. Why or why not?

**The planes each take 40 seconds to reach the intersection.  
So they arrive at the same time.**

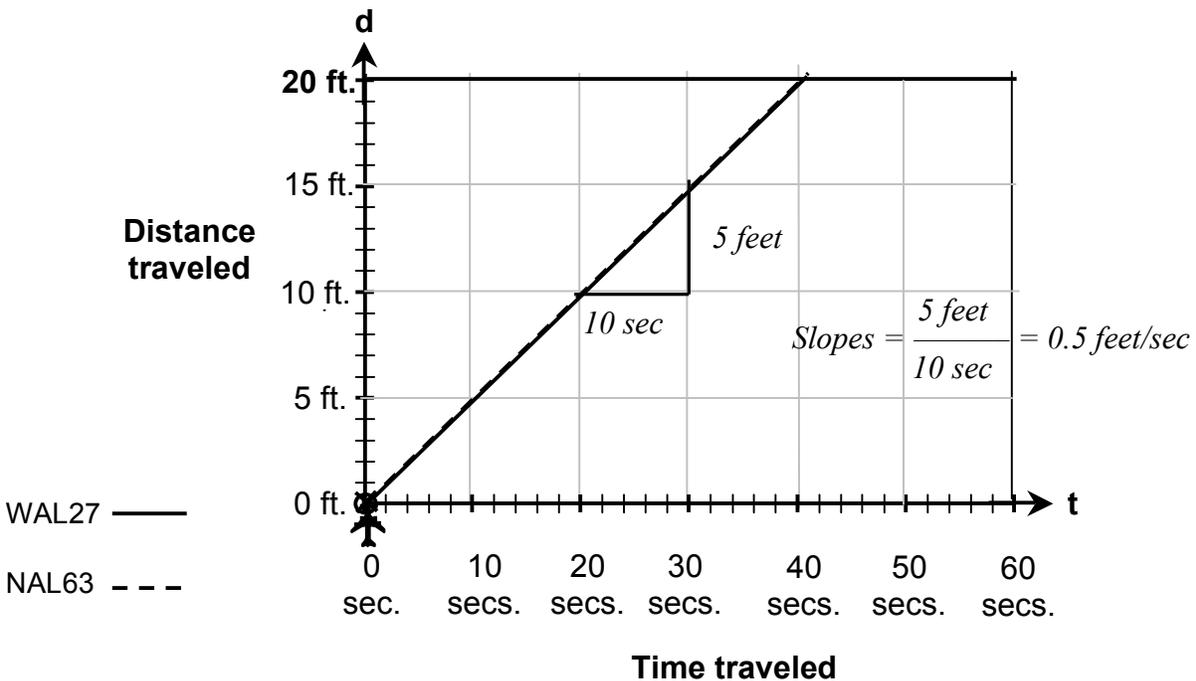
8. If you think two planes will meet, what would you tell the air traffic controller  
to do to avoid a collision?

**Change the speed or change the route of one of the planes.**


**Student Workbook: Do the Calculations—Graph Linear Equations (cont.)**

*Discussion: For each line, its slope represents the speed of the corresponding plane. Since the planes have the same speed, their lines have the same slope.*

*Furthermore, each plane has the same corresponding equation, so the lines coincide.*



9. Write the number that is the slope of the line representing WAL27. **0.5 feet/second**

10. Write the number that is the slope of the line representing NAL63. **0.5 feet/second**

11. What information does the slope of the line tell you about each plane?

**The slope of each line is 0.5 feet/second, the speed of the plane.**

**Student Workbook: Analyze Your Results**

*Discussion: Suppose two planes are traveling at the same constant (fixed) speed on two different routes and the planes are the same distance from the point where the two routes come together.*

*Since the planes are traveling at the same constant (fixed) speed and must each travel the same distance to the point of intersection, the planes will arrive at the intersection at the same time. So the planes will meet at the point where the routes come together.*

5. Are the planes' speeds the same or different? **Same**

6. Are the planes' starting distances the same or different? **Same**

7. Do the planes meet where the routes meet? **Yes**

Two planes are flying at the same speed on two different routes. The planes start at the same distance from the point where the routes meet.

11. The two planes will reach the point where the routes meet, at **the same time.**

**Posttest**

1. Do you think the planes will meet at the point where the routes intersect?

**Yes**

Why or why not? **The planes are traveling at the same constant speed and each plane must travel the same distance to the point where the routes intersect. So the planes will meet at the intersection.**

2. If not, how far apart do you think the planes will be when the first plane reaches the point where the routes intersect?

**0 feet**

Now consider this general problem.

Two planes are traveling at the same speed on two different routes.

Each plane is the same distance from the point where the two routes intersect.

3. Will the planes meet at the point where the routes intersect?

**Yes**

4. If you think two planes will meet, what could you tell the air traffic controller to do to avoid a collision? **Change the speed or change the route of one of the planes.**