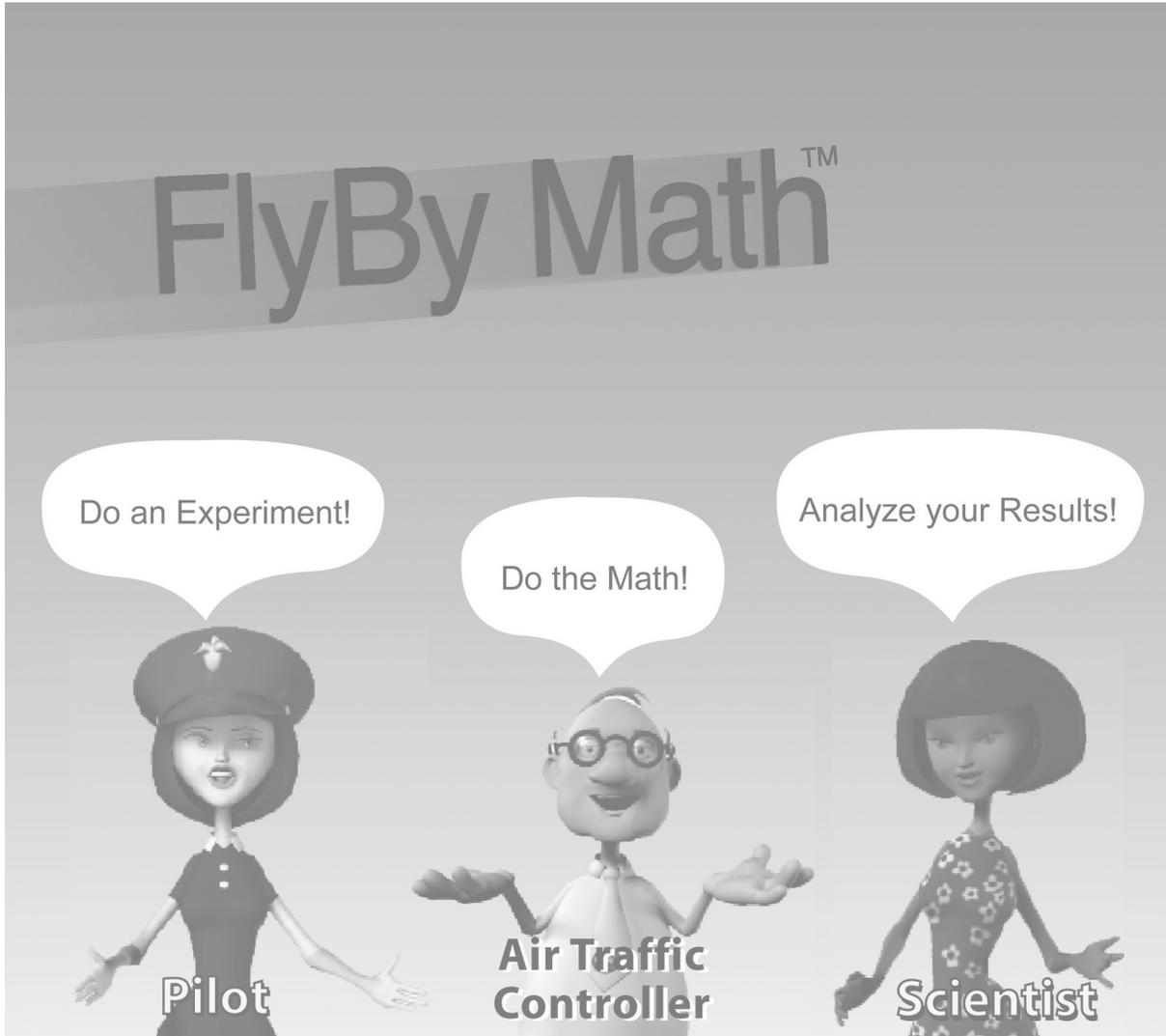


An Educator Guide to  
*FlyBy Math*<sup>TM</sup>  
Distance-Rate-Time Problems in Air Traffic Control  
Grades 5 – 9



**A Smart Skies<sup>TM</sup> Educational Product**

<http://www.nasa.gov>

<http://www.smartskies.nasa.gov/flyby>

<b>Educational Product</b>	
<b>Educators &amp; Students</b>	<b>Grades 5-9</b>
<b>EG-2004-11-111-ARC</b>	



*Smart Skies*<sup>™</sup> *FlyBy Math*<sup>™</sup> is available in electronic format through the NASA Education Portal—NASA’s electronic resources specifically developed for the educational community. This publication and other educational products may be accessed at the following address:

**<http://www.nasa.gov/audience/foreducators/topnav/materials/about/index.html>**

All *Smart Skies*<sup>™</sup> *FlyBy Math*<sup>™</sup> curriculum materials are free and available to download from the *FlyBy Math*<sup>™</sup> website:

**<http://www.smartskies.nasa.gov/flyby>**



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## A TEACHER INTRODUCTION TO FLYBY MATH™

### Overview of *FlyBy Math™*

### BECOMING ACQUAINTED WITH THE *FLYBY MATH™* DISTANCE-RATE-TIME PROBLEMS

*FlyBy Math™* consists of five Air Traffic Control (ATC) Problems. Each examines a different distance-rate-time air traffic scenario that an air traffic controller might encounter.

Each Air Traffic Control Problem features a Student Workbook containing an air traffic control experiment, paper-and-pencil calculations to support the experiment, and a student analysis of the experiment and calculations. The Workbook can be supplemented with optional pre- and post-tests. The *FlyBy Math™* curriculum materials also include video clips to introduce students to the nation's air traffic control system.

Each Air Traffic Control Problem is accompanied by a Teacher Guide with a full set of answers and solutions, as well as suggestions for implementing the specific airspace scenario.

The *FlyBy Math™* curriculum materials have been developed by NASA's Airspace Systems Program to engage students in Grades 5-9 in real-life applications of mathematics and science. The Airspace Systems program develops advanced computer-based systems to help pilots and air traffic controllers operate the nation's air transportation system with reduced flight delays and improved efficiency and access. Using the *FlyBy Math™* curriculum materials, your students learn to predict air traffic conflicts using distance, rate, and time relationships.

### Access to *FlyBy Math™*

All *FlyBy Math™* curriculum materials are free and available to download from the *FlyBy Math™* website:

**<http://www.smartskies.nasa.gov/flyby>**

### The 5 Air Traffic Control (ATC) Problems

In ATC Problems 1 through 4, two planes are flying at the same altitude on two jet routes that merge into one route. The controller must track the two planes to avoid a conflict. ATC Problem 5 addresses two airplanes on the same jet route, with the trailing airplane traveling faster than the leading airplane. In each Air Traffic Control Problem, each airplane travels at a constant (fixed) speed.



The Air Traffic Control Problems build upon one another and typically use the same sets of parameters (distances, speeds, and separation distances).

In each Air Traffic Control Problem, students:

- Assume the roles of pilots, air traffic controllers, and NASA scientists to conduct an experiment that simulates the airplane scenario.
- Assume the role of a NASA engineer and use guided paper-and-pencil activities to determine the number of seconds it takes each plane to travel a given distance along a jet route.

The five Air Traffic Control Problems are summarized in the following table.

<b>ATC Problems 1 – 4</b>		
<b>Two planes</b> are traveling on <b>different</b> jet routes. <b>Will they conflict</b> at the intersection of their routes? If so, which plane arrives first and when?		
ATC Problem	Plane Speeds	Distance from Intersection of Routes
1	Same	Same
2	Same	Different
3	Different	Same
4	Different	Different
<b>ATC Problem 5</b>		
<b>Two planes</b> are traveling on the <b>same</b> jet route. The following plane is going faster than the leading plane. <b>When will the planes conflict?</b>		
ATC Problem	Plane Speeds	Distance from End of Route
5	Different	Different

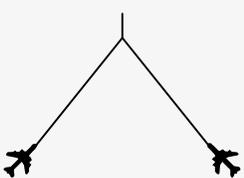
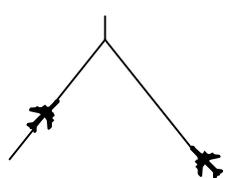


**Instructional Goals**

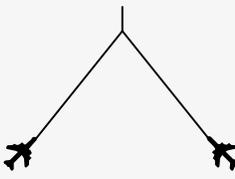
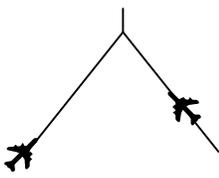
The *FlyBy Math™* curriculum materials have two overarching goals:

- To enable students to use mathematical reasoning and scientific inquiry to investigate and solve problems based on real-life scenarios.
- To offer students a variety of problem-solving tools and approaches, ranging from experiments to paper-and-pencil activities.

The following table lists the specific learning objectives for each *FlyBy Math™* air traffic control problem.

ATC Problem	Plane Speeds	Learning Objectives Students will determine...
<p style="text-align: center;"><b>1</b></p> 	<p style="text-align: center;">Same</p>	<ul style="list-style-type: none"> <li>• If two planes are traveling at the same constant (fixed) speed on two different routes and the planes are the same distance from the point where the two routes come together, the planes will arrive at the intersection at the same time. So the planes will meet at the point where the routes come together.</li> </ul>
<p style="text-align: center;"><b>2</b></p> 	<p style="text-align: center;">Same</p>	<ul style="list-style-type: none"> <li>• If two planes are traveling at the same constant (fixed) speed on two different routes and the planes are different distances from the point where the two routes come together, the planes will arrive at the intersection at different times. So the planes will not meet at the point where the routes come together.</li> <li>• Also, since the planes are traveling at the same constant (fixed) speed, the plane closest to the intersection will maintain its “headstart.” So at the intersection, the separation between the planes will be the same as the “headstart” of the plane that was closest to the intersection at the beginning of the problem.</li> </ul>



ATC Problem	Plane Speeds	Learning Objectives Students will determine...
<p style="text-align: center;"><b>3</b></p> 	Different	<ul style="list-style-type: none"> <li>If two planes are traveling at different constant (fixed) speeds on two different routes and the planes are each the same distance from the point where the two routes come together, the planes will arrive at the intersection at different times. So the planes will not meet at the point where the routes come together.</li> <li>Also, since the planes are traveling at different constant (fixed) speeds, their separation distance at the intersection is directly proportional to the difference in speeds. So, for example, if the difference in speeds were twice as great, then the separation at the intersection would also be twice as great.</li> </ul>
<p style="text-align: center;"><b>4</b></p> 	Different	<ul style="list-style-type: none"> <li>If two planes are traveling at different constant (fixed) speeds on two different routes and the planes are each a different distance from the point where the two routes come together, students must know those values in order to determine the separation distance between the planes at the intersection.</li> </ul>
<p style="text-align: center;"><b>5</b></p> 	Different	<ul style="list-style-type: none"> <li>If two planes are traveling at different speeds on the same route and the trailing plane is traveling faster than the leading plane, the trailing plane will close the gap at a rate equal to the difference in the speeds of the planes.</li> <li>So if the difference in speeds is twice as great and the starting distance between the planes remains the same as the original starting separation distance, then the trailing plane will close the gap at twice the original rate. Therefore, the amount of time for the trailing plane to catch up to the leading plane will be half as great.</li> <li>If the planes each travel at their original speeds but the starting distance between the planes is twice as great, then the trailing plane must now close twice the distance. However, the plane is traveling at the original rate. So the amount of time will double for the trailing plane to catch up to the leading plane.</li> </ul>



## Science Content Standards

*For a comprehensive alignment to the NSES, see **Appendix A** and **Appendix B** of this document.*

Each *FlyBy Math™* Air Traffic Control Problem supports several of the National Science Education Standards (NSES) for Grades 5-8 and 9-12.

In particular, the “Motions and Forces” Physical Science content standard is a key focus of each Air Traffic Control Problem:

*“As a result of their activities in grades 5-8 and 9-12, all students should develop an understanding of motions and forces.”*

The NSES cites this fundamental concept that underlies the standard:

*“(Grades 5-8) The motion of an object can be described by its position, direction of motion, and speed. That motion can be measured and represented on a graph.”*

To address this concept, the Air Traffic Control Problem activities employ a variety of representations and techniques including tables, one-dimensional graphs, two-dimensional graphs, and experimentation.

In addition to the Physical Science content standard, each *FlyBy Math™* Air Traffic Control Problem supports Science as Inquiry Content Standard A:

*“As a result of activities in grades 5-8 and 9-12, all students should develop abilities necessary to do scientific inquiry.”*

In particular, each Air Traffic Control Problem provides opportunities for students to:

- *“(Grades 5-8) Use appropriate tools and techniques to gather, analyze, and interpret data.”*
- *“(Grades 5-8) Develop description, explanation, predictions, and models using evidence.”*
- *“(Grades 5-8) Think critically and logically to make the relationships between evidence and explanations.”*
- *“(Grades 9-12) Use technology and mathematics to improve investigations and communication.”*

## Mathematics Content Standards

*For a comprehensive alignment to the NCTM Standards and Expectations, see **Appendix C**, **Appendix D**, and **Appendix E** of this document.*

Each *FlyBy Math™* Air Traffic Control Problem also supports many of the National Council of Teachers of Mathematics (NCTM) Standards and Expectations for Grades 5-9.

Particular emphasis is placed on content standards and expectations from Algebra (Grades 3-5, 6-8, and 9-12), Geometry (Grades 3-5), Measurement (Grades 6-8), and Data Analysis and Probability (Grades 3-5) as follows:

*(Algebra 3-5) Students “model problem situations with objects and use*



*representations such as graphs, tables, and equations to draw conclusions.”*

*(Algebra 3-5) Students “identify and describe situations with constant or varying rates of change and compare them.”*

*(Algebra 6-8) Students “model and solve contextualized problems using various representations, such as graphs, tables, and equations.”*

*(Algebra 6-8) Students “use graphs to analyze the nature of changes in quantities in linear relationships.”*

*(Algebra 9-12) Students “analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.”*

*(Geometry 3-5) Students “describe location and movement using common language and geometric vocabulary.”*

*(Geometry 3-5) Students “make and use coordinate systems to specify locations and to describe paths.”*

*(Geometry 3-5) Students “create and describe mental images of objects, patterns, and paths.”*

*(Measurement 6-8) Students “solve simple problems involving rates and derived measurements for such attributes as velocity and density.”*

*(Data Analysis and Probability 3-5) Students “collect data using observations, surveys, and experiments.”*

*(Data Analysis and Probability 3-5) Students “represent data using tables and graphs such as line plots, bar graphs, and line graphs.”*

## **Mathematics Process Standards**

Each Air Traffic Control Problem also supports the NCTM process standards (K-12) for Problem Solving, Communication, Connections, and Representation:

*(Problem Solving K-12) Students “solve problems that arise in mathematics and other contexts.”*

*(Problem Solving K-12) Students “apply and adapt a variety of strategies to solve problems.”*

*(Communication K-12) Students “communicate their mathematical thinking coherently and clearly to peers, teachers, and others.”*

*(Connections K-12) Students “recognize and apply mathematics in*

**Assessment**

*contexts outside of mathematics.”*

*(Representation K-12) Students “select, apply, and translate among mathematical representations to solve problems.”*

*(Representation K-12) Students “use representations to model and interpret physical, social, and mathematical phenomena.”*

Each *FlyBy Math™* Air Traffic Control Problem includes pre- and post-test assessment instruments that address the following learning objectives reflecting the national science and mathematics standards.

**Science**

Students will:

- Use data to construct an explanation of an airspace problem.
- Use evidence to put forth predictions, explanations, and models of an airspace problem.
- Measure and represent motion on a graph.

**Mathematics**

Students will:

- Solve an airspace problem involving rates of change.
- Model and solve an airspace problem using a variety of representations such as graphs, tables, and equations.
- Communicate their mathematical thinking.



## PREPARING TO TEACH *FLYBY MATH*<sup>TM</sup>

### Student and Teacher Materials

Each Air Traffic Control Problem features a Student Workbook containing an air traffic control experiment, paper-and-pencil calculations to support the experiment, and a student analysis of the experiment and calculations. The Workbook can be supplemented with optional pre- and post-tests. The *FlyBy Math*<sup>TM</sup> curriculum materials also include video clips to introduce students to the nation's air traffic control system.

Each Air Traffic Control Problem is accompanied by a Teacher Guide with a full set of answers and solutions, as well as suggestions for implementing the specific airspace scenario.

All *FlyBy Math*<sup>TM</sup> curriculum materials are free and available to download from the *FlyBy Math*<sup>TM</sup> website:

<http://www.smartskies.nasa.gov/flyby>

Visit the website to preview materials and print hard copies of the worksheets and other files.

### Selecting Materials

To prepare to teach *FlyBy Math*<sup>TM</sup>:

1. Decide which Air Traffic Control Problem you wish to teach. If you and your students are new to *FlyBy Math*<sup>TM</sup>, you may want to begin with one of the following Problems:

- ATC Problem 1: Two planes on merging jet routes; planes have same speeds; planes are same distance from the merge.
- ATC Problem 2: Two planes on merging jet routes; planes have same speeds; planes are different distances from the merge.

Many teachers have found ATC Problem 2 to be an appropriate starting point for their students.

See the table at the beginning of this document for a detailed summary of ATC Problems 1 through 5.

2. Decide which mathematics calculation method you want your students to use.

To help you make this choice, read the description of each method in the following Implementation section of this document.

*If you have never used FlyBy Math<sup>TM</sup>, you may find it helpful to preview each of the calculation worksheets for the ATC Problem you have selected.*



### Downloading Materials

*If you do not want to print the documents, you can simply preview them online.*

### Reviewing Materials

3. Go to the *FlyBy Math™* website:

<http://www.smarskies.nasa.gov/flyby>

4. Follow the online instructions to select and download the materials.
5. With the student and teacher materials in hand, read the following Implementation section in this document.
6. Assemble the materials for the classroom experiment. (See the experiment materials list in the following Implementation section.)
7. Duplicate the student materials to provide each student with one copy.
8. Now you're cleared for takeoff! ✈️



## IMPLEMENTING *FLYBY MATH*<sup>TM</sup> WITH YOUR STUDENTS

### Sequence of Instructional Activities

*As you read this section, you may find it helpful to have a copy of a Student Workbook and teacher materials.*

*You may choose to spread the experiment and calculation activities over two or three class periods.*

*You can choose which paper-and-pencil math activity to assign.*

### Video

Estimated time:  
30 minutes

*The video clips are available on the FlyBy Math<sup>TM</sup> website:*

*<http://www.smartskies.nasa.gov/flyby>*

The following sequence of instructional activities is recommended for implementing each *FlyBy Math*<sup>TM</sup> Air Traffic Control Problem:

1. If your students are new to *FlyBy Math*<sup>TM</sup>, begin with the video introduction to air traffic control.
2. (Optional) Administer the Pretest.
3. Assign the Read the Problem activity from the Student Workbook.
4. Have your students Set Up and Do the Experiment using those worksheets in the Student Workbook.
5. Assign the Calculation activity you have selected to guide your students through the mathematics that supports the experiment.
6. Assign the Analyze Your Results activity from the Student Workbook to help your students compare their experimental results with their calculations.
7. Assign the Extension problem, available for Air Traffic Control Problems 3, 4, and 5 only.
8. (Optional) Administer the Post-test.

Each of these eight instructional activities is described in greater detail as follows.

### See Air Traffic Controllers on the Job

The video clips (produced by NASA and the FAA) introduce students to the federal air traffic control system, to the controllers who operate the system, and to the tools the controllers use.

The video clips feature a wealth of information about airplanes and air traffic control including engaging animations, discussions with real air traffic controllers, and visits to air traffic control facilities.

The video viewing may be conducted as an individual, a small-group, or a whole-class activity.

### Materials

Video clips available online



PC or Mac

**Assessment:  
Pretest  
(Optional)**

Estimated time:  
15 - 30 minutes

*Instead of distributing the pretest, you may want to use the questions to guide a classroom discussion.*

**Pretest—Make a Prediction**

The pretest steps the student through a careful reading of the airplane problem statement. The student is then asked to predict the outcome of the given airplane scenario.

The pretest may be assigned as either an individual or a small-group activity.

If your students have completed other *FlyBy Math™* Air Traffic Control Problems, you may want to direct them to use a particular calculation method or methods to answer the pretest questions. The assessment package includes a worksheet with a blank vertical line plot and a grid that students can use as they do the mathematics.

**Materials**

Assessment Package Worksheets: Pretest—Make a Prediction  
Lines and Grid (optional)

**Workbook:  
Problem Statement**

Estimated time:  
10 minutes

**Read the Problem**

This worksheet steps the student through a careful reading of the airplane problem statement. The basic air traffic control data are presented in both standard and metric measurements. Several of the calculation methods require students to use the distance traveled by each plane in 10 seconds. So this worksheet asks students to calculate that distance.

**Workbook:  
Experiment**

Estimated time:  
Setup—30 minutes  
Experiment—30 minutes

**Classroom Experiment**

In this small-group activity, students set up the experiment by marking off the jet routes on the classroom floor, on butcher paper, or on an outdoor area. To conduct the experiment, students assume the roles of pilots, air traffic controllers, and NASA scientists. The pilots step down the jet route at a prescribed pace. The NASA scientists track and record the pilots' distances from the intersection of the routes (ATC Problems 1-4) or distances along the jet route (ATC Problem 5). The air traffic controllers set the pace and record the time when the first plane arrives at the intersection (ATC Problems 1-4) or when the trailing plane overtakes the leading plane (ATC Problem 5).

You may want to give students an overview of the experiment including an explanation of what they will do in each activity.

You may also want to ask your students to compare the experiment distances and speeds with the real-world speeds given in the Teacher Guide for each ATC Problem.



*You may want to ask your students to estimate the route layout before they measure.*

*Students who have little experience in measurement may benefit from first practicing skip counting (e.g., counting by 6 to prepare them to measure 6-inch lengths).*

*It may be difficult for some student pilots to take 6-inch (or 4-inch or 3-inch) steps by placing one foot in front of the other. Instead, advise the pilots to place one foot on either side of the jet route and align their toes at each mark. The NASA scientist can assist by pointing to the next mark. It may be helpful for students to practice.*

### Set Up the Experiment

Air Traffic Control Problems 1-4 require two jet routes. If there is not enough room to set up two routes (e.g., a 20-foot route and a 16-foot route) at right angles to one another, another angle may be used. As an alternative, the routes may be set up parallel to each other. (Caution: parallel routes may confuse students who have not had much experience with the experiment. They may not make the connection between the parallel routes and the given merging routes.) In any case allow enough distance between the routes so that the two pilots are not distracted by one another.

If your classroom has 1-foot by 1-foot floor tiles, your students can use the tiles as guidelines for placing masking tape at the appropriate intervals along the jet route.

In place of masking tape, you may want to lay out the routes with reusable materials such as cashier's tape or butcher paper. Then you can roll up and store the routes for another experiment. If you do so, have students make sure they have the correct number of markings (i.e., they have not lost or gained any markings) when they lay out the routes for the follow-up experiment.

You may want to set up one pair of jet routes as a model that your students can copy.

After a group of students has completed its jet route set-up, you may find it helpful to have them compare their work with another student set-up.

### Conduct the Experiment

Assign students to positions on 6-8 person teams as follows:

- Lead Air Traffic Controller (1 student)
- Secondary Air Traffic Controller (1 student) (Problems 2, 3, and 4 only)
- Pilots (2 students)
- NASA Scientists, 1 or 2 for each plane (2 – 4 students)

After the jet routes are set up, have one group of students demonstrate the experiment while the rest of the class observes. Discuss and address any issues that may arise.

Perform the activity at least three times. Compare the results of each trial. Discuss the validity of the results. Then have your students use the results of their three experiments to choose the best answer to this question:



- (ATC Problems 1 – 4) What is the separation distance between the planes when the first pilot reaches the point where the routes meet?
- (ATC Problem 5) How long will it take the trailing plane to catch up with the leading plane?

### Experiment Enhancements (Optional):

1. Repeat the activity using different students as the Air Traffic Controllers, Pilots, and NASA Scientists.
2. Repeat the activity using longer jet routes. Keep the ratio of the lengths the same. Increase the plane speed by increasing the step size.
3. Have students draw a scale model of the experiment using real-world data found in the Teacher Guide for each Air Traffic Control Problem.

### Materials

- Student Workbook Worksheet: Set Up and Do the Experiment

#### Other materials:

- sidewalk chalk or masking tape or cashier's tape or a knotted rope
- measuring tape or ruler
- marking pens (optional)
- 1 stopwatch or 1 watch with a sweep second hand or 1 digital watch that indicates seconds
- pencils
- signs identifying pilots, controllers, and NASA scientists  
Note: the signs are available on the *FlyBy Math™* website.
- clipboard (optional)

### Workbook: Math Calculations

Estimated time:  
15 - 30 minutes

*You will choose just one or two of these methods to incorporate in the Student Workbook. If you wish to assign more than two calculation methods, assign an additional problem to avoid repetition within one problem.*

### Calculate the Time for Each Plane

This activity presents six different methods students can use to determine the number of seconds:

- for each plane to arrive at the point where their routes merge (Air Traffic Control Problems 1-4)
- for the trailing plane to catch up with the leading plane (Air Traffic Control Problem 5).

The calculation methods increase in order of difficulty as follows:

- Counting feet and seconds using a jet route diagram
- Drawing blocks to make a bar graph
- Plotting points on two vertical lines
- Plotting points on a Cartesian coordinate system



- Deriving and using the distance-rate-time formula
- Graphing two linear equations

This table indicates the range of appropriate grade levels for each problem and method:

		Grade:	5	6	7	8	9
<b>Math Method:</b>							
	Counting		√	√			
	Blocks		√	√	√		
	Vertical Line Graph		√	√	√	√	
	Cartesian Plot			√	√	√	√
	Formula				√	√	√
	Graphing Two Equations					√	√
<b>Problem:</b>							
	Same Speed	1	√	√	√		
		2	√	√	√	√	√
	Different Speeds	3		√	√	√	√
		4			√	√	√
		5				√	√

*For a detailed discussion of each method, see the Answers and Solutions in the Teacher Guide for each ATC Problem.*

Each method is presented on a separate worksheet.

Each worksheet may be assigned as either an individual or a small-group activity.

You may choose to start your students with the Vertical Line Graph worksheet. If your students need more support, you can choose the Counting worksheet or the Blocks worksheet. If your students need a greater challenge, you can choose the Cartesian Plot worksheet, the Formula worksheet, or the Graphing Two Equations worksheet.

Each method is described briefly as follows.

### Count Feet and Seconds

Students use patterns and skip-counting to step their way down a jet route diagram and solve the problem. At the end of this activity, students may realize it is faster to multiply to obtain the answers.

Prerequisite skills: skip-counting by 2's and/or 3's (depending upon the problem)

**Draw Blocks**

For each plane, students draw blocks, each representing the distance the plane travels in 10 seconds. The students “stack” their blocks along a vertical number line that represents the plane’s jet route.

For each plane, students begin to stack the blocks at the plane’s starting point located at the bottom of a vertical line.

To help students make the connection between the physical problem and the block graph, students also plot the corresponding points on a jet route diagram.

Prerequisite skills: read and build a bar graph with a vertical scale marked in 1-foot units; count by 10s.

**Plot Points on Vertical Lines**

This graph is similar to the way families record and compare the height of their children at the same ages. They mark off each child’s birthday height (distance from the floor) on a doorway and then record their age (time since birth) beside the height mark.

For each plane, students plot their points along a vertical number line that represents the plane’s jet route.

The bottom of the number line represents the starting point of the plane.

To help students make the connection between the physical problem and the vertical line graph, students also plot the corresponding points on a jet route diagram.

Prerequisite skills: plot a point on a (vertical) number line.

**Plot Points on a Cartesian Coordinate System Grid**

The points are plotted in the first quadrant with “time (in seconds)” on the horizontal axis and “distance traveled (in feet)” on the vertical axis.

To help students make the connection between the physical problem and the graph, students also plot the corresponding points on a jet route diagram.

Prerequisite skills: plot a point on a Cartesian coordinate system (the xy-plane)



### **Derive and Use the Distance-Rate-Time Formula**

First, students use patterns to derive the distance-rate-time formula in the form  $d = rt$ .

Then students apply the distance-rate-time formula in the form  $t = d/r$ . (Problems 1-4 only)

Prerequisite skills:

Use patterns to make a generalization.

Substitute numbers into a formula.

### **Graph Two Linear Equations**

For each plane, the distance-rate-time formula,  $d = rt$ , is used to obtain a linear equation that describes the distance traveled by the plane as a function of time traveled.

The points are plotted in the first quadrant with “time (in seconds)” on the horizontal axis and “distance traveled (in feet)” on the vertical axis.

Students are also asked to find the slope of each line and interpret that number in the context of the airspace problem.

Prerequisite skills:

Graph a linear equation by making a table of ordered pairs.

Find the slope of a line given the equation of the line and the graph of the line.

Enhancement (optional):

You may want to ask your students to find the  $y$ -intercepts of each line and interpret those  $y$ -intercepts in the context of the airspace problem.

For each plane, the  $y$ -intercept represents the plane’s initial distance from the beginning of the jet route.

### **Materials**

*Choose one* of the following Calculation Worksheets to remain in the Student Workbook. (Remove the other Calculation Worksheets.)

- Do the Calculations—Count Feet and Seconds
- Do the Calculations—Draw Blocks
- Do the Calculations—Plot Points on Lines
- Do the Calculations—Plot Points on a Grid
- Do the Calculations—Use the Distance-Rate-Time Formula
- Do the Calculations—Graph Two Linear Equations


**Workbook:  
Analysis**

Estimated time:  
45 minutes

**Compare the Experimental Results with the Predicted Results**

This activity may be assigned as either an individual or a small-group activity.

After your students have completed the experiment and a calculation worksheet, the Analyze Your Results worksheet helps them compare the results of the experiment with the results of their calculations. If the results are different, you may want to ask the students why the experiment and their calculations do not match.

As part of the analysis, you may also want to ask your students to create a similar problem in a different setting. For example, your students might create a similar problem involving two cars (rather than two planes).

The analysis also guides students through a generalization of the given airspace scenario.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would *not* be appropriate because a launched rocket typically accelerates and therefore its speed is not constant.)

**Materials**

-- Student Workbook Worksheet: Set Up and Do the Experiment

**Workbook:  
Extension**

Estimated time:  
15 minutes

*Extensions are available for Air Traffic Control Problems 2, 3, 4, and 5, only.*

Each extension introduces a separation requirement at the point where the routes intersect. For safety reasons, when the first plane reaches the intersection, the planes must be separated by a distance greater than or equal to a given standard separation distance. If their separation is less than this standard, a separation violation will occur.

Students are asked to review their calculations to determine the separation distance between the planes at the intersection of the routes. They are then asked whether that distance meets the separation requirement.

**Materials**

-- Student Workbook Worksheet: Extension  
(Available for ATC Problems 2, 3, 4, and 5, only.)

**Assessment:  
Posttest  
(Optional)**

Estimated time:  
15 - 30 minutes

**Posttest**

This activity may be assigned as either an individual or a small-group activity.

If your students have completed other *FlyBy Math™* Air Traffic Control



Problems, you may want to direct them to use a particular calculation method or methods to answer the posttest questions. The assessment package includes a worksheet with a blank vertical line plot and a grid that students can use as they do the mathematics.

**Materials**

Assessment Package Worksheets: Posttest  
Lines and Grid (optional)

**PROVIDING NASA WITH YOUR EVALUATION**

In order to ensure that these educational materials are the best they can be for teachers across the nation, NASA needs your feedback and suggestions.

Please take a few minutes to fill out and mail the enclosed *Educator Reply Card* found in the back of this document, or respond online at the Internet address found on the card.

Thank you!



## APPENDICES

### Appendix A

#### Alignment of *FlyBy Math™* to the National Science Education Standards (NSES) for Grades 5-8

Science Content Standard	<i>FlyBy Math™</i> Activity
<b>Science as Inquiry</b>	
Design & conduct a scientific investigation.	--Conduct simulation and measurement for several aircraft conflict problems.
Use appropriate tools and techniques to gather, analyze, and interpret data.	--Conduct simulation and measurement for several aircraft conflict problems.
Develop descriptions, explanations, predictions, and models using evidence.	--Use calculations and experimental evidence to predict, describe, and explain several aircraft conflict problems.
Think critically and logically to make the relationships between evidence and explanations.	--Compare predictions, calculations, and experimental evidence for several aircraft conflict problems.
<b>Physical Science</b>	
<b>Motion and Forces</b>	
As a result of their activities in grades 5-8, all students should develop an understanding of motions and forces.	
The motion of an object can be described by its position, direction of motion, and speed. That motion can be measured and represented on a graph.	--Calculate and measure the position and time of simulated aircraft. Represent that motion using tables, graphs, equations, and experimentation.

#### Alignment of *FlyBy Math™* to the National Science Education Standards (NSES) for Grades 9-12

Science Content Standard	<i>FlyBy Math™</i> Activity
<b>Science as Inquiry</b>	
Use technology and mathematics to improve investigations and communications.	--Conduct simulation and measurement for several aircraft conflict problems. --Use formulas and graphs to solve and analyze aircraft conflict problems and to communicate results.
<b>Physical Science</b>	
<b>Motion and Forces</b>	
As a result of their activities in grades 9-12, all students should develop an understanding of motions and forces.	--Calculate and measure the position and time of simulated aircraft. Represent that motion using tables, graphs, equations, and experimentation.



## Appendix B

### Alignment of *FlyBy Math™* to the National Council of Teachers of Mathematics (NCTM) Standards and Expectations for Grades 3-5

<b>Mathematics Standard &amp; Expectations</b>	<b><i>FlyBy Math™</i> Activity</b>
<b>Algebra</b>	
Use mathematical models to represent and understand quantitative relationships.	
Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.	--Represent distance, speed, and time relationship for constant speed cases using tables, bar graphs, line graphs, equations, and a Cartesian coordinate system.  --Use tables, bar graphs, line graphs, equations, and a Cartesian coordinate system to draw conclusions.
Identify and describe situations with constant or varying rates of change and compare them.	--Compare airspace scenarios for both the same and different starting conditions and the same and different rates.
<b>Geometry</b>	
Specify locations and describe spatial relationships using coordinate geometry and other representational systems.	
Describe location and movement using common language and geometric vocabulary.	--Explain and justify solutions regarding the motion of two airplanes using the results of plotting points on a schematic of a jet route, on a vertical line graph, and on a Cartesian coordinate system.
Make and use coordinate systems to specify locations and to describe paths.	--Plot points on a schematic of a jet route, on a vertical line graph, and on a Cartesian coordinate system to describe the motion of two airplanes.
Use visualization, spatial reasoning, and geometric modeling to solve problems.	
Create and describe mental images of objects, patterns, and paths.	--Predict the relative motion of two airplanes on given paths.
<b>Data Analysis and Probability</b>	
Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.	
Collect data using observations, surveys, and experiments.	--Conduct a simulation of each airplane scenario.
Represent data using tables and graphs such as line plots, bar graphs, and line graphs.	--Represent distance, rate, and time data using line plots, bar graphs, and line graphs.



<b>Problem Solving</b>	
Solve problems that arise in mathematics and other contexts.	--Apply mathematics to predict and analyze aircraft conflicts and validate through experimentation.
Apply and adapt a variety of strategies to solve problems.	--Use tables, graphs, and equations to solve aircraft conflict problems.
<b>Communication</b>	
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.	--Predict outcomes and explain results of mathematical models and experiments.
<b>Connections</b>	
Recognize and apply mathematics in contexts outside of mathematics.	--Apply mathematics to solving distance, rate, and time problems for aircraft conflict scenarios.
<b>Representation</b>	
Select, apply, and translate among mathematical representations to solve problems.	--Choose among tables, bar graphs, line graphs, a Cartesian coordinate system, and equations to model aircraft conflicts and predict outcomes.
Use representation to model and interpret physical, social, and mathematical phenomena.	--Use tables, bar graphs, line graphs, a Cartesian coordinate system, and equations to model aircraft conflicts and predict outcomes.



## Appendix C

### Alignment of *FlyBy Math™* to the National Council of Teachers of Mathematics (NCTM) Standards and Expectations for Grades 6-8

<b>Mathematics Standard &amp; Expectations</b>	<b><i>FlyBy Math™</i> Activity</b>
<b>Algebra</b>	
Use mathematical models to represent and understand quantitative relationships.	
Model and solve contextualized problems using various representations, such as graphs, tables, and equations.	--Represent distance, speed, and time relationship for constant speed cases using tables, bar graphs, line graphs, equations, and a Cartesian coordinate system.  --Use tables, bar graphs, line graphs, equations, and a Cartesian coordinate system to draw conclusions.
Analyze change in various contexts.	
Use graphs to analyze the nature of changes in quantities in linear relationships.	--Use graphs to compare airspace scenarios for both the same and different starting conditions and the same and different constant (fixed) rates.
<b>Measurement</b>	
Apply appropriate techniques, tools, and formulas to determine measurements.	
Solve simple problems involving rates and derived measurements for such attributes as velocity and density.	--Use the distance-rate-time formula to predict and analyze aircraft conflicts.
<b>Problem Solving</b>	
Solve problems that arise in mathematics and other contexts.	--Apply mathematics to predict and analyze aircraft conflicts and validate through experimentation.
Apply and adapt a variety of strategies to solve problems.	--Use tables, graphs, and equations to solve aircraft conflict problems.
<b>Communication</b>	
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.	--Predict outcomes and explain results of mathematical models and experiments.
<b>Connections</b>	
Recognize and apply mathematics in contexts outside of mathematics.	--Apply mathematics to solving distance, rate, and time problems for aircraft conflict scenarios.
<b>Representation</b>	
Select, apply, and translate among mathematical representations to solve problems.	--Choose among tables, bar graphs, line graphs, a Cartesian coordinate system, and equations to model aircraft conflicts and predict outcomes.
Use representation to model and interpret physical, social, and mathematical phenomena.	--Use tables, bar graphs, line graphs, a Cartesian coordinate system, and equations to model aircraft



Mathematics Standard & Expectations	FlyBy Math™ Activity
	conflicts and predict outcomes.

## Appendix D

### Alignment of *FlyBy Math™* to the National Council of Teachers of Mathematics (NCTM) Standards and Expectations for Grades 9-12

Mathematics Standard & Expectations	FlyBy Math™ Activity
<b>Algebra</b>	
Understand patterns, relations, and functions.	
Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.	--Represent distance, speed, and time relationship for constant speed cases using linear equations and a Cartesian coordinate system.  --Interpret the slope of a line in the context of a distance-rate-time problem.
<b>Problem Solving</b>	
Solve problems that arise in mathematics and other contexts.	--Apply mathematics to predict and analyze aircraft conflicts and validate through experimentation.
Apply and adapt a variety of strategies to solve problems.	--Use tables, graphs, and equations to solve aircraft conflict problems.
<b>Communication</b>	
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.	--Predict outcomes and explain results of mathematical models and experiments.
<b>Connections</b>	
Recognize and apply mathematics in contexts outside of mathematics.	--Apply mathematics to solving distance, rate, and time problems for aircraft conflict scenarios.
<b>Representation</b>	
Select, apply, and translate among mathematical representations to solve problems.	--Choose among tables, bar graphs, line graphs, a Cartesian coordinate system, and equations to model aircraft conflicts and predict outcomes.
Use representation to model and interpret physical, social, and mathematical phenomena.	--Use tables, bar graphs, line graphs, a Cartesian coordinate system, and equations to model aircraft conflicts and predict outcomes.



## GLOSSARY

<b>Aircraft</b>	Device(s) that are used or intended to be used for flight in the air, and when used in air traffic control terminology, may include the flight crew.
<b>Air Traffic</b>	Aircraft operating in the air or on an airport surface, exclusive of loading ramps and parking areas.
<b>Air Traffic Control</b>	A service operated by appropriate authority to promote the safe, orderly and expeditious flow of air traffic.
<b>Conflict Resolution</b>	The resolution of potential conflicts between aircraft that are radar identified and in communication with ATC by ensuring that radar targets do not touch.
<b>Flight Plan</b>	Specified information relating to the intended flight of an aircraft that is filed orally or in writing with an FSS or an ATC facility.
<b>Flight Path</b>	A line, course, or track along which an aircraft is flying or intended to be flown.
<b>Route</b>	A defined path, consisting of one or more courses in a horizontal plane, which aircraft traverse over the surface of the earth.
<b>Separation</b>	In air traffic control, the spacing of aircraft to achieve their safe and orderly movement in flight and while landing and taking off.



## NASA RESOURCES FOR EDUCATORS

### **FlyBy Math™ Website**

The *FlyBy Math™* website contains links to additional educational materials related to Airspace Systems.

<http://www.smartskies.nasa.gov/flyby>

### **CORE**

#### **Central Operation of Resources for Educators**

**(CORE)** was established for the national and international distribution of NASA-produced educational materials in multimedia format. Educators can obtain a catalogue and an order form by one of the following methods:

CORE

Lorain County Joint Vocational School  
15181 Route 58 South  
Oberlin, OH 44074-9799

Toll Free Ordering Line: 1-866-776-CORE

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E-mail [nasaco@leeca.org](mailto:nasaco@leeca.org)

Home Page: <http://core.nasa.gov>

### **Educator Resource Center Network**

#### **Educator Resource Center Network (ERCN)**

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[http://www.education.nasa.gov/about/contacts/Educator\\_Resource\\_Center\\_Network.html](http://www.education.nasa.gov/about/contacts/Educator_Resource_Center_Network.html)

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NASA Educator Resource Center  
**NASA Ames Research Center**  
Mail Stop 253-2  
Moffett Field, CA 94035-1000  
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One Tranquility Base  
Huntsville, AL 35807  
Phone: (256) 544-5812

*CA*

NASA Educator Resource Center for  
**NASA Jet Propulsion Laboratory**  
Village at Indian Hill  
1460 East Holt Avenue, Suite 20  
Pomona, CA 91767  
Phone: (909) 397-4420

*VA and MD's Eastern Shores*

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**GSFC/Wallops Flight Facility**  
Visitor Center Building J-17  
Wallops Island, VA 23337  
Phone: (757) 824-2298

**NASA Johnson Space Center**

1601 NASA Road One  
Houston, TX 77058  
Phone: (281) 244-2129

*KY, NC, SC, VA, WV*

Virginia Air & Space Center  
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**NASA Langley Research Center**  
600 Settlers Landing Road  
Hampton, VA 23669-4033  
Phone: (757) 727-0900 x 757

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Stennis Space Center, MS 39529-6000  
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<http://www.nasa.gov/audience/foreducators/topnav/subjects/about/index.html>

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Check the Internet for programs listings at:

<http://www.nasa.gov/ntv>

For more information on NTV, contact:

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## Smart Skies™ FlyBy Math™ Educator Guide Distance-Rate-Time Problems in Air Traffic Control

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Number of Students:

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Number of Others:

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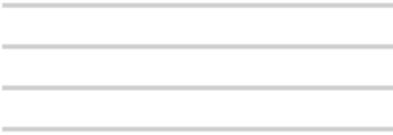
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