Two planes on merging routes are:
-- traveling at the same speed.
An alternate route is not available.

LineUp With Math™

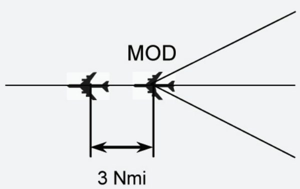
Math-Based Decisions in Air Traffic Control for Grades 5 - 9

Problem Set E

Resolving 2-Plane Traffic Conflicts by Changing Speed

Teacher Guide with Answer Sheets

Overview of Problem Set E



Goal

Estimated class time: 1 to 2 hours

In this Problem Set, students will determine whether two planes traveling on different merging routes will line up with proper spacing at MOD (the last intersection before the planes leave the airspace sector). If the spacing is not adequate, students will change the speed of one plane to achieve the proper spacing at MOD.

The planes are traveling at the same altitude and the same constant (fixed) speeds.

In *LineUp With Math*™, this is the first set of problems where students use speed change to achieve safe and proper spacing.

This Problem Set also introduces an optimal solution time for each Simulator problem. A "target time" is posted on the Simulator screen. This target is the minimum time required for the last plane to reach the intersection at MOD. An on-screen clock keeps track of the flight time for a student's solution.

Each problem can be explored with the interactive Air Traffic Control (ATC) Simulator. Five of the problems can be more closely examined with Student Workbook E (print-based). The Workbook provides a structured learning environment for exploring the problems with paper-and-pencil worksheets that introduce students to pertinent air traffic control concepts as well as problem analysis and solution methods.

Objectives

Each plane is traveling at 600 knots, the maximum speed allowed. So to resolve a spacing conflict, students must reduce a plane's speed.

Students will:

- Analyze a sector diagram to identify a spacing conflict between two planes, each traveling at the same speed.
- Resolve the spacing conflict by changing the speed of one plane.
- (Optional) Learn that a given percent reduction in plane speed yields the same percent reduction in distance traveled in the original amount of time. (For a mathematical derivation of this relationship, see Appendix III.)

Prerequisites

Before attempting the current Problem Set, it is *strongly* recommended that students complete Problem Set A that introduces essential air traffic control vocabulary, units, and representations.

It is also recommended that students complete Problem Set D that introduces students to the effects of changes in speed.

Materials

- ATC Simulator (web-based)
- Student Workbook E (print-based)

The materials are available on the *LineUp With Math*™ website:

http://www.smartskies.nasa.gov/lineup

A separate student website gives students easy access to the Simulator only (and not to the answers and solutions provided on the teacher website):

http://www.atcsim.nasa.gov

ATC Simulator

A complete description of the ATC Simulator is contained in the Educator Guide for LineUp With Math™.

For a Simulator quick start guide and an animated tutorial, visit the LineUp With Math™ website.

Interactive Air Traffic Control Simulator

Students can explore Problem Set E with the interactive ATC Simulator. Each problem features a 2-plane conflict that can be resolved by a speed change.

The Simulator problems for Problem Set E are:

2-4*; 2-5*; 2-6*; 2-7*; 2-8*; 2-11; 2-12; 2-13; 2-14; 2-15; 2-16

Problems with an asterisk (*) are supported by worksheets in Student Workbook E.

The optimal solution time ("target time") is displayed on the screen for each Simulator problem. This time is the minimum required for the last plane to reach the intersection at MOD. An on-screen clock keeps track of the flight time for a student's solution.

For a complete set of answers and solutions to all Problem Set E Simulator problems, see Appendix I of this document.

For a discussion of the key points associated with the first five Simulator problems, see the worksheet notes in the following Student Workbook section.

Student Workbook

It is recommended that you have a copy of Workbook E open while you read these notes.

The worksheet title is the same as the associated Simulator problem.

The Student Workbook consists of five worksheets, one for each of the five featured Simulator problems listed below.

Simulator Problem	Worksheet Title
2-4*	Problem 2-4
2-5*	Problem 2-5
2-6*	Problem 2-6
2-7*	Problem 2-7
2-8*	Problem 2-8

Each problem features a spacing conflict with different starting conditions. As students progress through the worksheets, they likely will require less guidance and structure, and the worksheets reflects this.

For a complete set of answers to each worksheet, see Appendix II of this document.

For each worksheet, the key points are briefly described as follows.

Worksheet: *Problem 2-4*

*In the sector diagram, each route flows only **towards** MOD. E.g., a plane may fly from MINAH to OAL, but cannot fly from OAL to MINAH.*

- After students identify the spacing conflict at MOD, they determine it will take each plane 3 minutes to arrive at MOD. After students decrease the speed of one plane, the faster plane will still take 3 minutes to arrive at MOD. So, the planes will fly 3 minutes before Ideal Spacing must be achieved.
- To resolve the spacing conflict, students begin by reducing the speed of one plane by 60 knots. (Either plane can be selected since neither has a headstart.) At the reduced speed, this plane will travel 1 nautical mile less each minute.
- Finally, students apply the 1 nautical per minute distance reduction for 3 minutes to achieve Ideal Spacing (3 nautical miles) exactly at MOD.

Worksheet: *Problem 2-5*

- Using the same problem-solving approach as in Problem 2-4, students determine it will take 2 minutes for each plane to arrive at MOD. After students decrease the speed of one plane, the faster plane will still take 2 minutes to arrive at MOD. A single 60-knot speed decrease will achieve only a 2 nautical mile spacing in 2 minutes at MOD. This is less than Ideal Spacing at MOD.
- To resolve the spacing conflict, students must make a 120-knot speed decrease (the equivalent of **two** 60-knot decreases).
- This results in 4 nautical mile spacing at MOD, which is greater than Ideal Spacing. The students are asked to suggest a way to achieve Ideal Spacing at MOD. This requires increasing the slower plane's speed to the same speed (600 knots) as the leading plane as soon as Ideal Spacing is achieved.
- In the next problem, Problem 2-6, students will be given the opportunity to make such a speed increase.

Worksheet: *Problem 2-6*

- Students use the same problem-solving approach as in Problem 2-4. However, unlike Problems 2-4 and 2-5, one plane has a headstart (1 nautical mile). For the trailing plane, a single 60-knot speed decrease will result in more than Ideal Spacing at MOD.
- The 3 nautical mile Ideal Spacing is achieved before MOD. As soon as this Ideal Spacing is achieved, the trailing plane's speed should be increased to the same speed as the leading plane. This will maintain Ideal Spacing all the way to MOD and beyond.
- Students are asked to specify the number of minutes (2 minutes) after which they will speed up the trailing plane. This is the number of minutes at which Ideal Spacing will be achieved.

Worksheet: ***Problem 2-7***

In this problem, students work with decimals.

- Students use the same problem-solving approach as in Problem 2-6. However, in the current problem, both planes pass through OAL before they arrive at MOD. So students must check for Minimum Separation (2 nautical miles) at OAL as well as for Ideal Spacing of 3 nautical miles at MOD.
- First, students check for Ideal Spacing at MOD. This is because the goal is to have Ideal Spacing at MOD. After the students have determined the strategy to achieve this goal, they next check to see if their strategy violates Minimum Separation at OAL. (If it does, they must change their initial strategy to resolve the violation at OAL.)
- Since each plane is 25 nautical miles from MOD, they will arrive at MOD at the same time. Since there is **no** spacing between the planes at MOD, this does not meet the Ideal Spacing goal (3 nautical miles).
- To resolve the spacing conflict, students begin by reducing the speed of one plane by 60 knots. (Either plane can be selected since neither has a headstart.) At the reduced speed, this plane will travel 1 nautical mile less each minute.
- The faster plane takes 2.5 minutes to travel 25 nautical miles to MOD at 600 knots. In 2.5 minutes, with a 60-knot speed reduction, the slower plane will fall behind 2.5 nautical miles (2.5 minutes x 1 nautical mile/minute = 2.5 nautical miles). This is less than Ideal Spacing at MOD.
- To achieve at least Ideal Spacing at MOD, a 120-knot speed decrease is required. This speed decrease will yield a 5 nautical mile spacing at MOD (2.5 minutes x 2 nautical miles/minute = 5 nautical miles).
- Before the planes reach MOD, they will each pass through OAL. So students must check for Minimum Separation at OAL. Each plane starts 15 nautical miles from OAL. The faster plane takes 1.5 minutes to travel 15 nautical miles to OAL. A 120-knot speed decrease (2 nautical miles per minute) will result in a 3 nautical mile separation at OAL (1.5 minutes x 2 nautical miles/minute = 3 nautical miles). This meets the Minimum Separation requirement of at least 2 nautical miles. This also provides Ideal Spacing at OAL.
- To maintain the 3 nautical mile Ideal Spacing all the way to MOD and beyond, students must speed up the slower plane exactly at OAL. (Note: Students made similar calculations in Problem 2-6.)

Worksheet (Optional): ***Understand the % Method***

- A plane, traveling at its original speed, can cover a certain distance in a given amount of time. If the plane's speed is reduced by a certain percent, then in the given amount of time, the distance covered is reduced by the same percent. (For a mathematical derivation of this relationship, see Appendix III.)
- This percent relationship is especially easy to apply in the *LineUp With Math*™ problems since all speed reductions are done in increments of 10% of the original speed. In particular, the original plane speed is always 600 knots and the original speed is always reduced in 60-knot increments. (Note: 60 is 10% of 600.) With a 10% speed reduction, the distance traveled is also reduced by 10% (of the original distance). To find 10% of the original distance, students need only divide the distance by 10, that is, they need only move the decimal point one place to the left.
- Note that with this percent method, students do *not* need to calculate the amount of time it will take the lead plane to reach MOD (as they have done in previous worksheets).

Worksheet (Optional): ***Problem 2-8***

- In this problem, students are guided through the percent method introduced in the previous worksheet.
- The leading plane starts 20 nautical miles from MOD. The trailing plane starts 21 nautical miles from MOD. In the time it takes the leading plane to travel 20 nautical miles to MOD, the trailing plane will also travel 20 nautical miles. So to calculate the % decrease in travel distance for the trailing plane, students must use 20 nautical miles (**not** 21) for the distance traveled.

Answer Sheets

For a set of answers and solutions to all Simulator problems, visit the LineUp With Math™ website.

Answer sheets for each of the Problem Set E Simulator problems can be found in Appendix I of this document.

Answer sheets for each worksheet in Student Workbook E can be found in Appendix II of this document.

A mathematical derivation of the Percent Method can be found in Appendix III of this document.

LineUp With Math™

APPENDIX I

Air Traffic Control Simulator

Simulator Solutions for Problem Set E

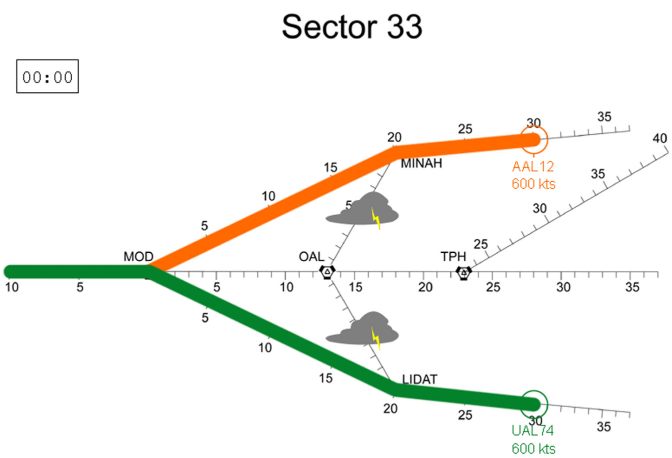
**2-4*, 2-5*, 2-6*, 2-7*, 2-8*,
2-11, 2-12, 2-13, 2-14, 2-15, 2-16**

Problems with an asterisk (*) are supported by worksheets in Student Workbook E

Problem 2-4

Solution

Starting Conditions:



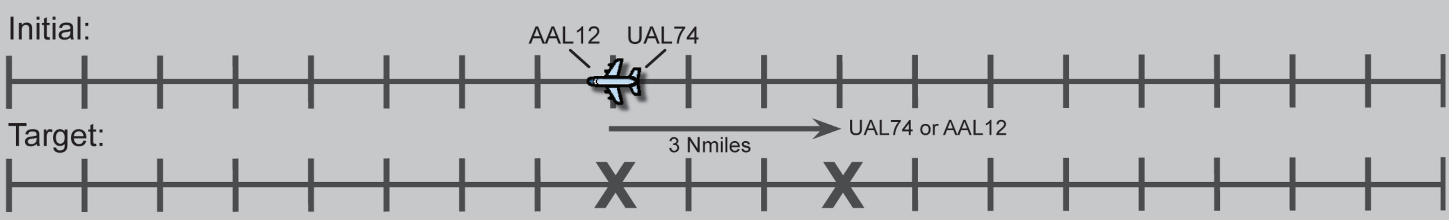
Plane	From	Through	To	Distance	Speed
AAL12	MINAH		MOD	30	600
UAL74	LIDAT		MOD	30	600

- Route from **LIDAT** to **OAL is closed**.
- Route from **MINAH** to **OAL is closed**.
- Ideal spacing at **MOD** is 3 nautical miles**.**

Analysis:

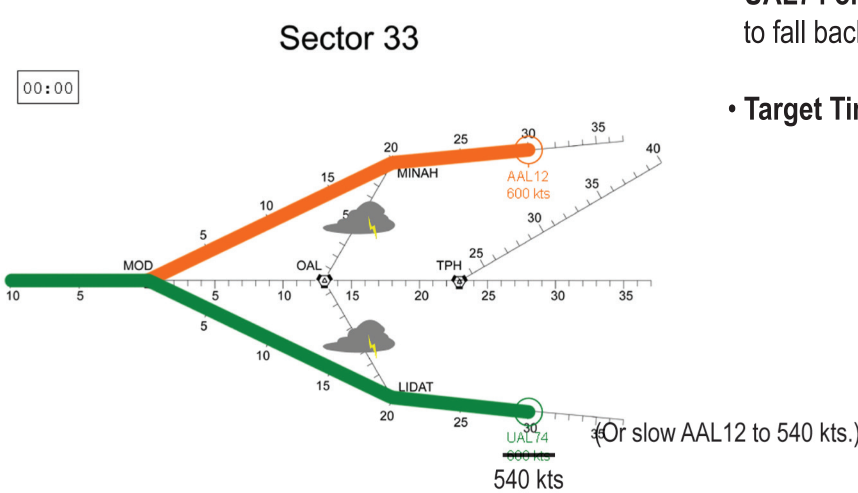
- **<u>Conflict:</u> AAL12 <u>AND</u> UAL74** will arrive at MOD at the same time.
- **Weather** prevents **AAL12** or **UAL74** from rerouting.
- **UAL74** or **AAL12** need to slow down to fall back 3 nautical miles.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	AAL12	30	0
1st	UAL74	30	



Solution:

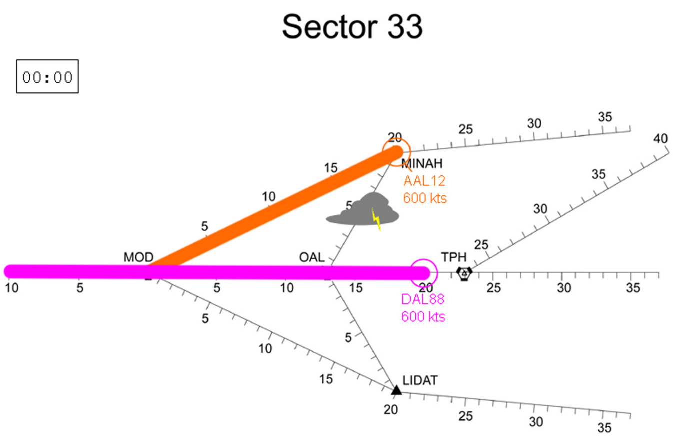
- **UAL74 or AAL12** - Slow down to 540 knots for 3 minutes to fall back 3 nautical miles. Then speed up to 600 knots.
- **Target Time** - 3 minutes and 18 seconds.



Problem 2-5

Solution

Starting Conditions:



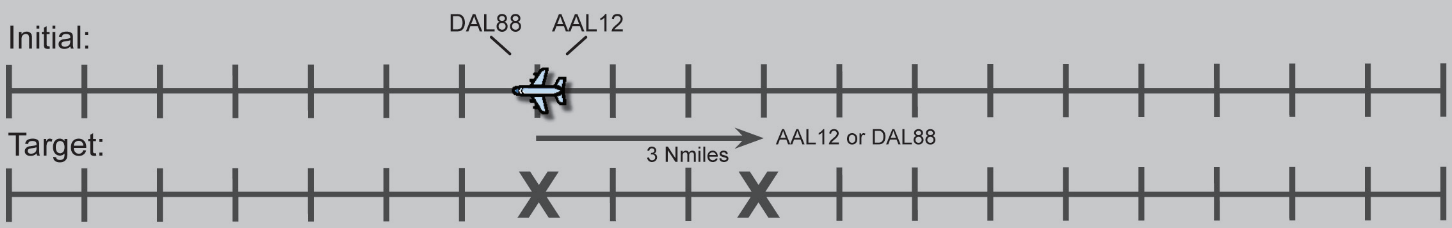
Plane	From	Through	To	Distance	Speed
AAL12	MINAH		MOD	20	600
DAL88	OAL		MOD	20	600

- Route from **MINAH** to **OAL is closed**.
- Ideal spacing at **MOD** is 3 nautical miles**.**

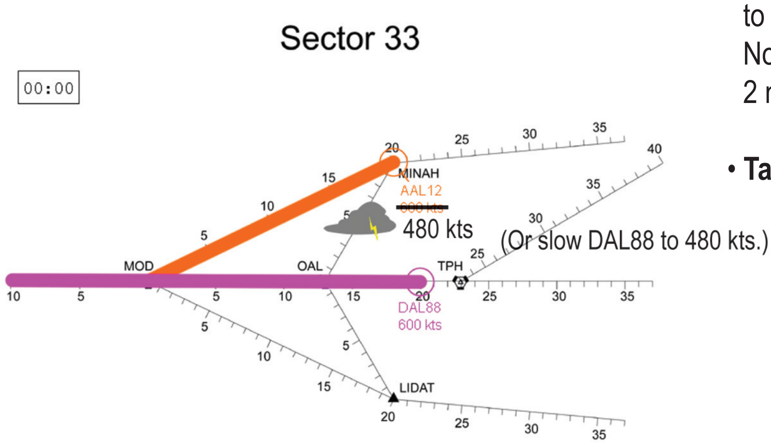
Analysis:

- **<u>Conflict:</u> DAL88 <u>AND</u> AAL12** will arrive at MOD at the same time.
- Weather prevents AAL12 from rerouting.
- **AAL74** or **DAL88** need to slow down to fall back 3 nautical miles.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	DAL88	20	0
1st	AAL12	20	



Solution:



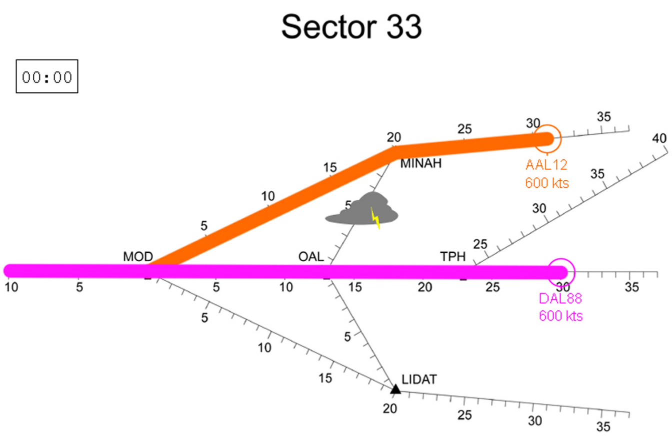
- **AAL12 or DAL88** - Slow down to 480 knots for 1.5 minutes to fall back 3 nautical miles. Then speed up to 600 knots. Note: Slowing to 540 knots would only result in falling back 2 nautical miles in the 20 nautical miles to MOD.

- **Target Time** - 2 minutes and 18 seconds.

Problem 2-6

Solution

Starting Conditions:



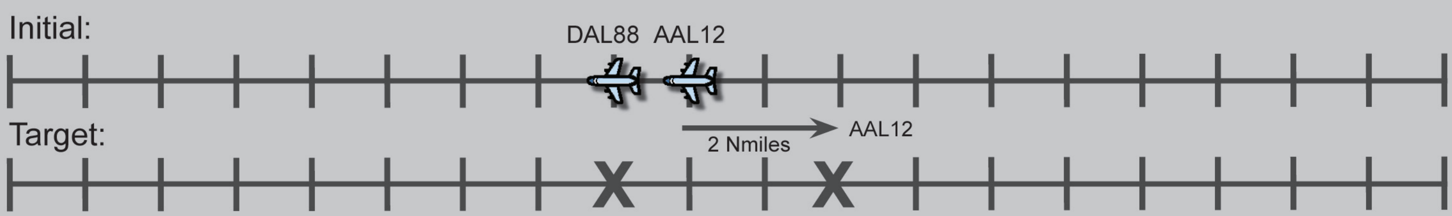
Plane	From	Through	To	Distance	Speed
AAL12	MINAH		MOD	31	600
DAL88	TPH	OAL	MOD	30	600

- Route from **MINAH** to **OAL is closed**.
- Ideal spacing at **MOD** is 3 nautical miles**.**

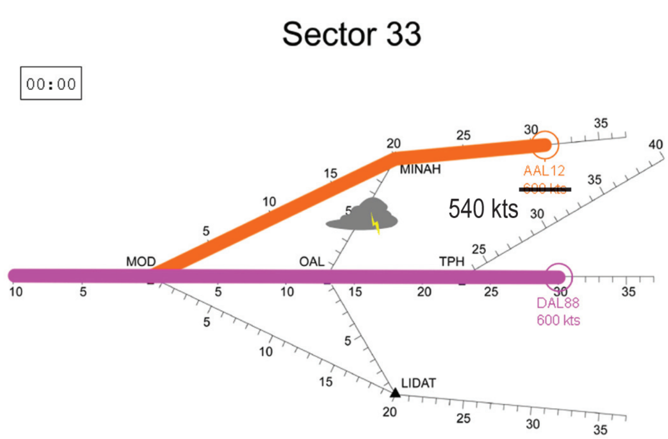
Analysis:

- **<u>Conflict:</u> AAL12** will arrive at MOD **1 nautical mile** behind **DAL88**.
- **Weather** prevents **AAL12** from rerouting.
- **AAL12** needs to slow down to fall back 2 nautical miles.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	DAL88	30	1
2nd	AAL12	31	



Solution:

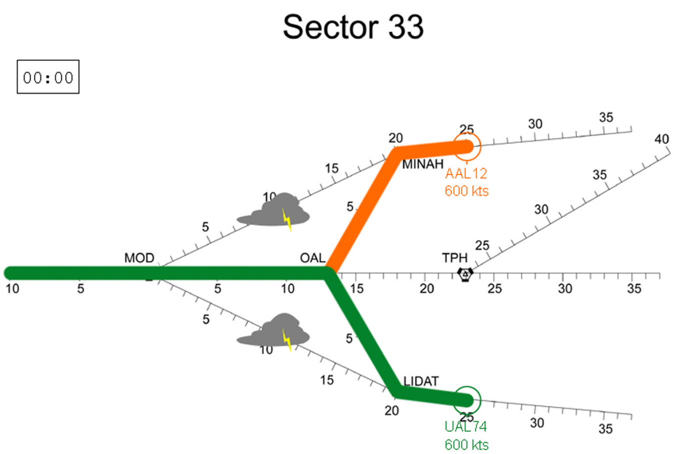


- **AAL12** - Slow down to 540 knots for 2 minutes to fall back 2 nautical miles. Then speed up to 600 knots.
- **Target Time** - 3 minutes and 18 seconds.

Problem 2-7

Solution

Starting Conditions:



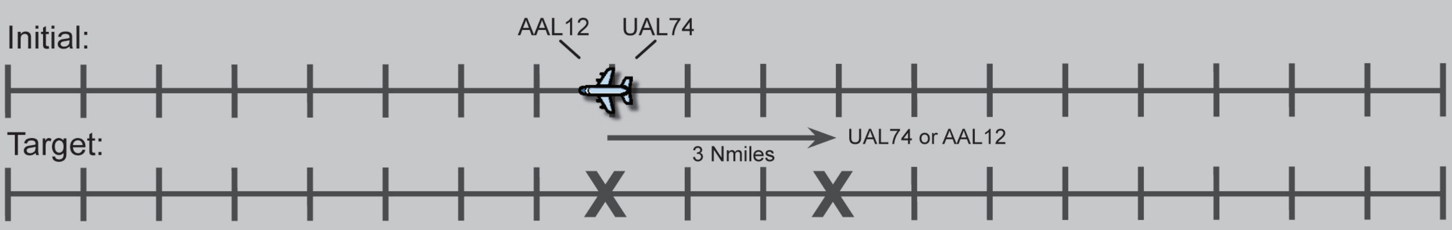
Plane	From	Through	To	Distance	Speed
AAL12	MINAH	OAL	MOD	28	600
UAL74	LIDAT	OAL	MOD	28	600

- Route from **MINAH** to **MOD is closed**.
- Route from **LIDAT** to **MOD is closed**.
- Ideal spacing at **MOD** is 3 nautical miles**.**

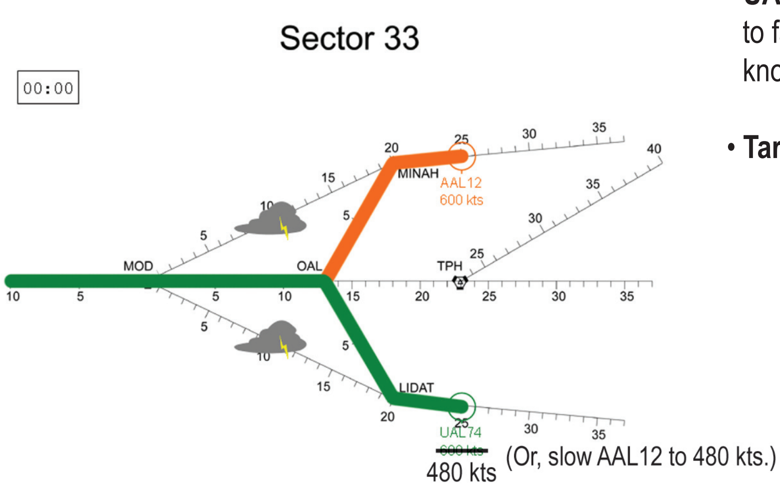
Analysis:

- **<u>Conflict:</u> AAL12 <u>AND</u> UAL74** will arrive at OAL at the same time.
- **Weather** prevents **AAL12 <u>AND</u> UAL74** from rerouting.
- **UAL74** or **AAL12** need to slow down to fall back 2 nautical miles by OAL **and** 3 nautical miles by MOD.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	AAL12	28	→ 0
1st	UAL74	28	



Solution:



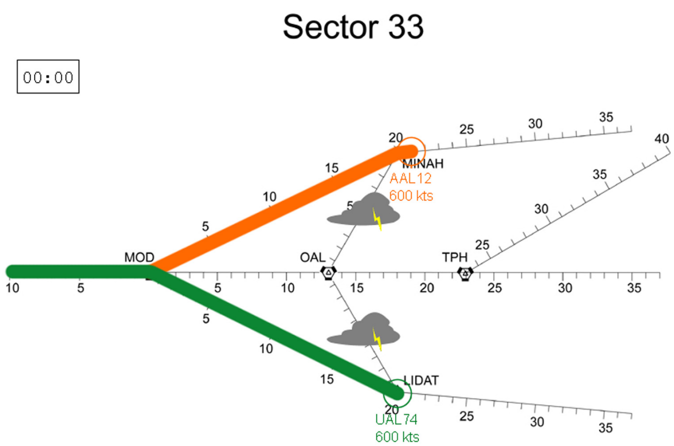
- **UAL74 or AAL12** - Slow down to 480 knots for 1.5 minutes to fall back 3 nautical miles **at OAL**. Then speed up to 600 knots.

- **Target Time** - 3 minutes and 6 seconds.

Problem 2-8

Solution

Starting Conditions:



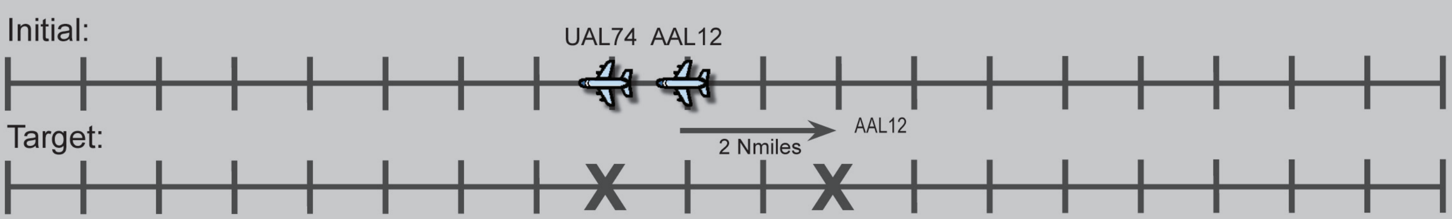
Plane	From	Through	To	Distance	Speed
AAL12	MINAH		MOD	21	600
UAL74	LIDAT		MOD	20	600

- Route from **LIDAT** to **OAL is closed**.
- Route from **MINAH** to **OAL is closed**.
- Ideal spacing at **MOD** is 3 nautical miles**.**

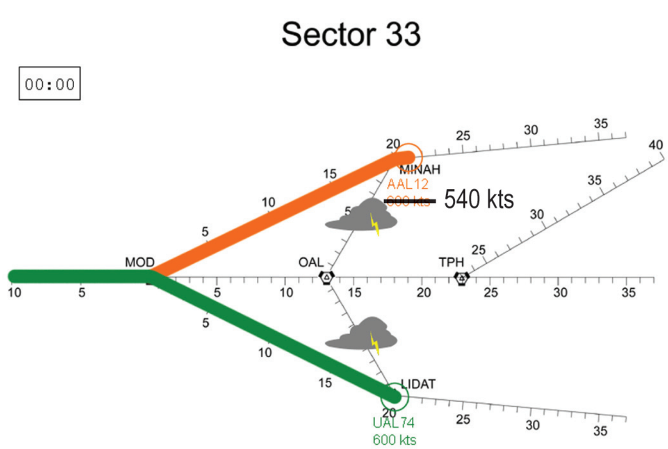
Analysis:

- **<u>Conflict:</u> AAL12** will arrive at MOD **1 nautical mile** behind **UAL74**.
- **Weather** prevents UAL74 or AAL12 from rerouting.
- **AAL12** needs to slow down to fall back 2 nautical miles.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	UAL74	20	1
2nd	AAL12	21	



Solution:

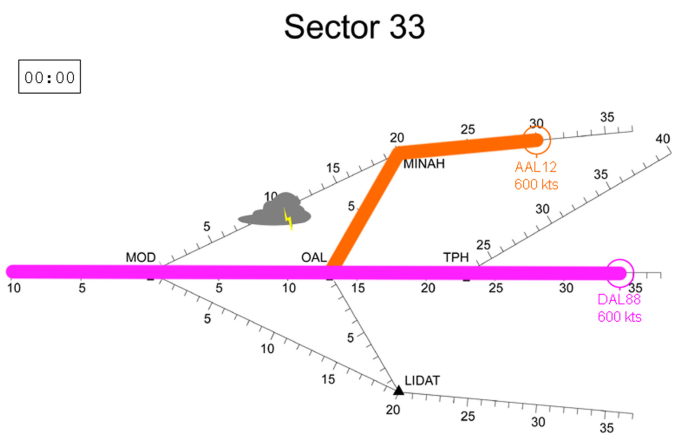


- **AAL12** - Slow down to 540 knots for 2 minutes to fall back 2 nautical miles. Then speed AAL12 up to 600 knots.
- **Target Time** - 2 minutes and 18 seconds.

Problem 2-11

Solution

Starting Conditions:



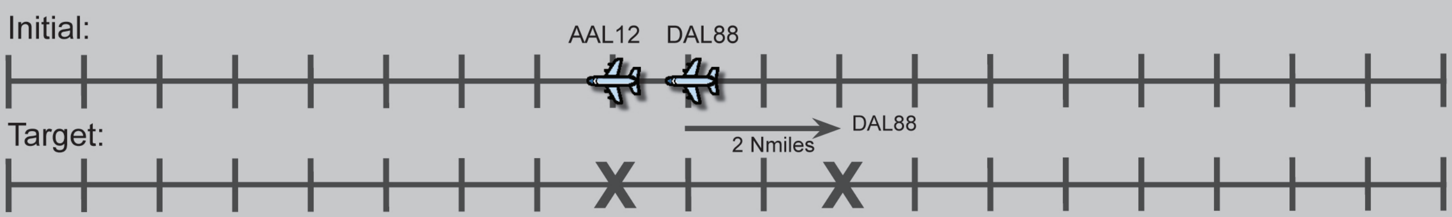
Plane	From	Through	To	Distance	Speed
AAL12	MINAH	OAL	MOD	33	600
DAL88	TPH	OAL	MOD	34	600

- Route from **MINAH** to **MOD is closed**.
- Ideal spacing at **MOD** is 3 nautical miles**.**

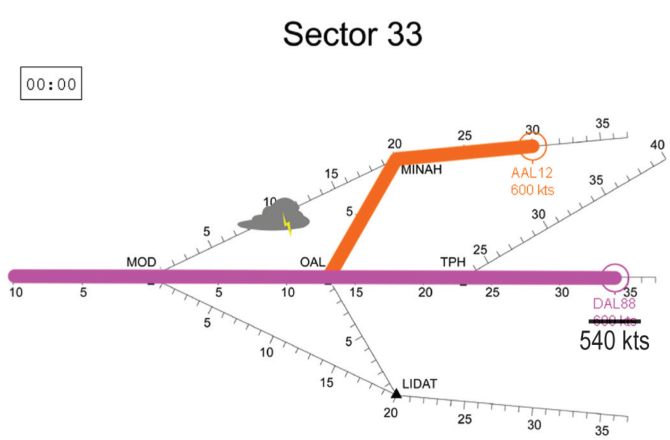
Analysis:

- **Conflict: DAL88** will arrive at OAL **1 nautical mile** behind **AAL12**.
- **Weather** prevents **AAL12** from rerouting.
- **DAL88** needs to slow down to fall back 2 nautical miles by **MOD** (and at least 1 nautical mile by OAL).

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	AAL12	33	1
2nd	DAL88	34	



Solution:



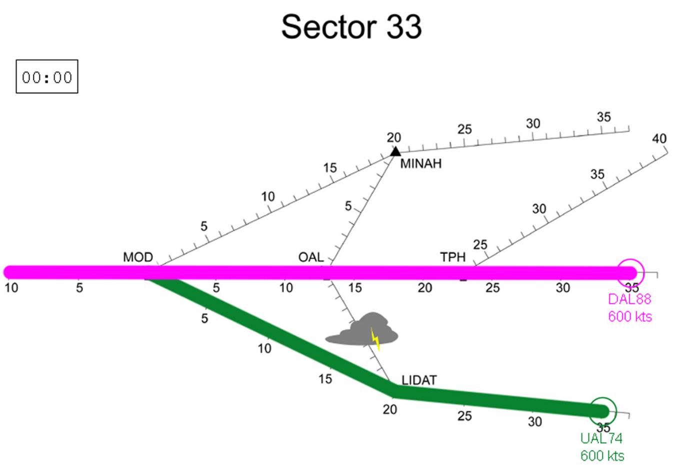
- **DAL88** - Slow down to 540 knots for 2 minutes to fall back 2 nautical miles. Then speed up to 600 knots.

- **Target Time** - 3 minutes and 36 seconds.

Problem 2-12

Solution

Starting Conditions:



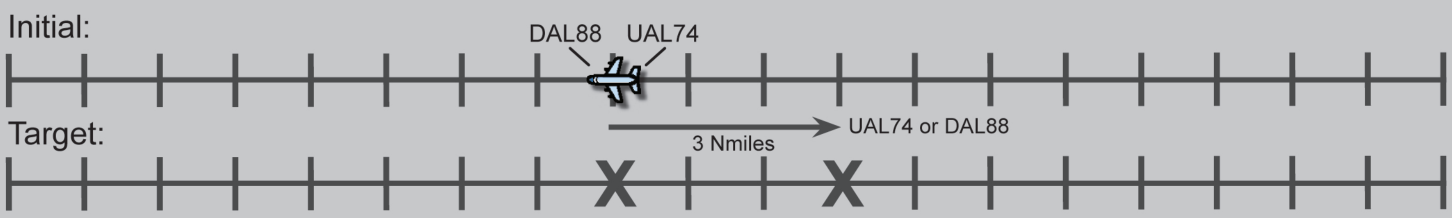
Plane	From	Through	To	Distance	Speed
DAL88	TPH	OAL	MOD	35	600
UAL74	LIDAT		MOD	35	600

- Route from **LIDAT** to **OAL is closed**.
- Ideal spacing at **MOD** is 3 nautical miles.

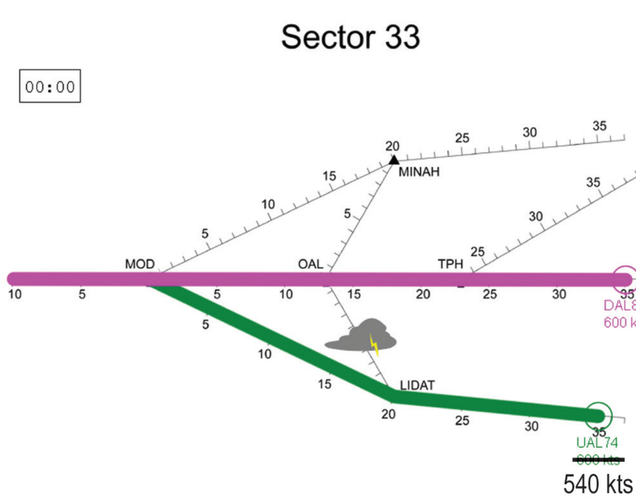
Analysis:

- **<u>Conflict:</u> DAL88 <u>AND</u> UAL74** will arrive at MOD at the same time.
- **Weather** prevents **UAL74** from rerouting.
- **UAL74** or **DAL88** need to slow down to fall back 3 nautical miles.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	DAL88	35	0
1st	UAL74	35	



Solution:



(Or slow DAL88 to 540 kts.)

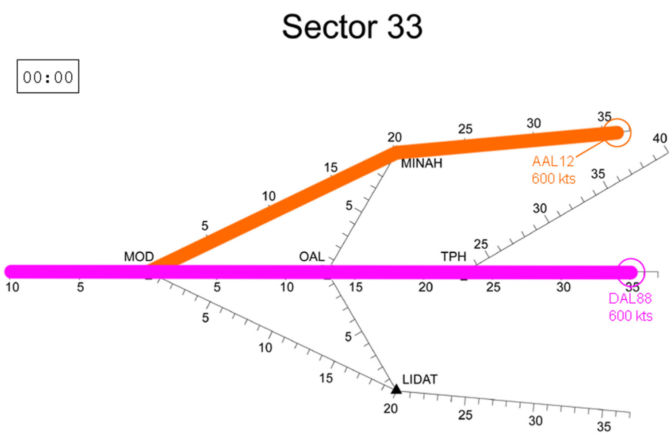
- **UAL74 or DAL88** - Slow down to 540 knots for 3 minutes to fall back 3 nautical miles. Then speed up to 600 knots.

- **Target Time** - 3 minutes and 48 seconds.

Problem 2-13

Solution

Starting Conditions:



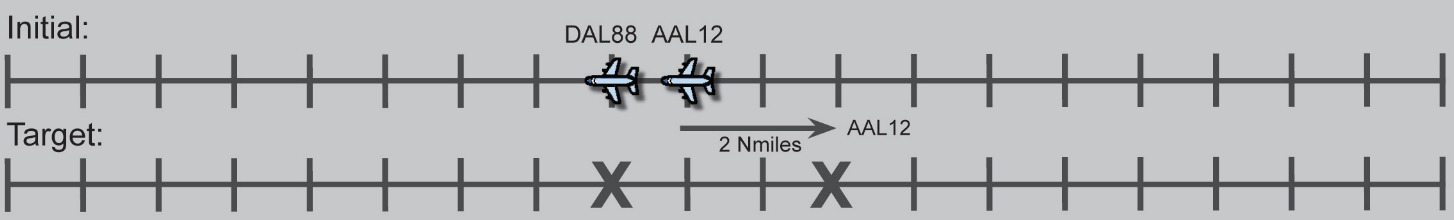
Plane	From	Through	To	Distance	Speed
AAL12	MINAH		MOD	36	600
DAL88	TPH	OAL	MOD	35	600

• Ideal spacing at **MOD** is 3 nautical miles**.**

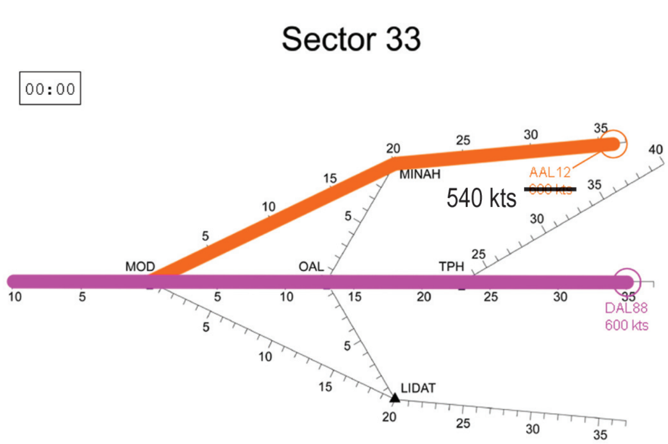
Analysis:

• **Conflict: AAL12** will arrive at MOD **1 nautical mile** behind **DAL88**.
• **AAL12** needs to slow down to fall back 2 nautical miles.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	DAL88	35	1
2nd	AAL12	36	



Solution:



• **AAL12** - Slow down to 540 knots for 2 minutes to fall back 2 nautical miles. Then speed up to 600 knots.

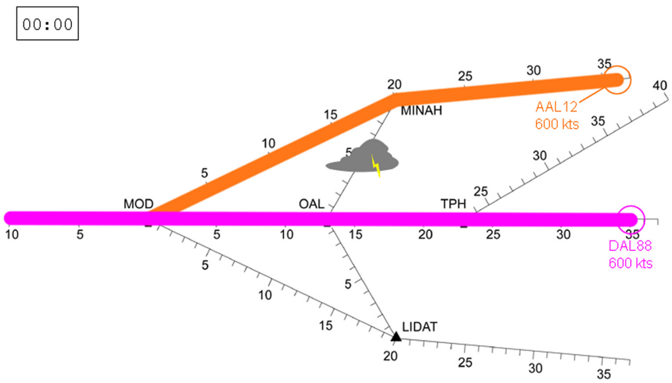
• **Target Time** - 3 minutes and 48 seconds.

Problem 2-14

Solution

Starting Conditions:

Sector 33



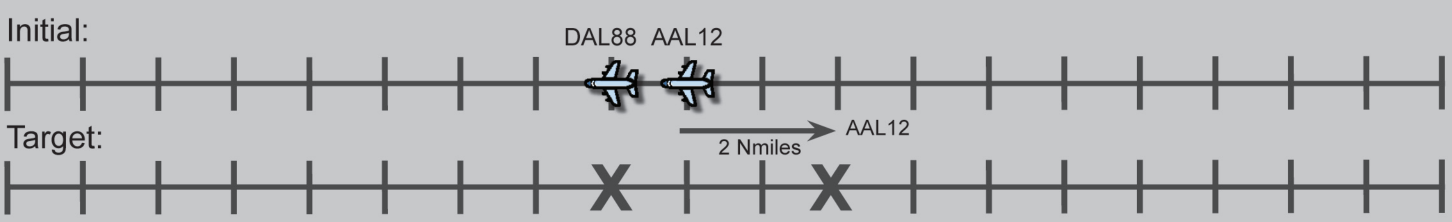
Plane	From	Through	To	Distance	Speed
AAL12	MINAH		MOD	36	600
DAL88	TPH	OAL	MOD	35	600

- Route from **MINAH** to **OAL is closed**.
- Ideal spacing at **MOD** is 3 nautical miles**.**

Analysis:

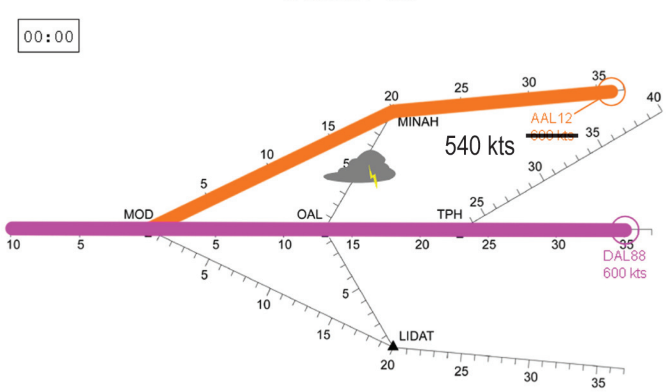
- __Conflict:__ **AAL12** will arrive at MOD **1 nautical mile** behind **DAL88**.
- **Weather** prevents **AAL12** from rerouting.
- **AAL12** needs to slow down to fall back 2 nautical miles.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	DAL88	35	1
2nd	AAL12	36	



Solution:

Sector 33

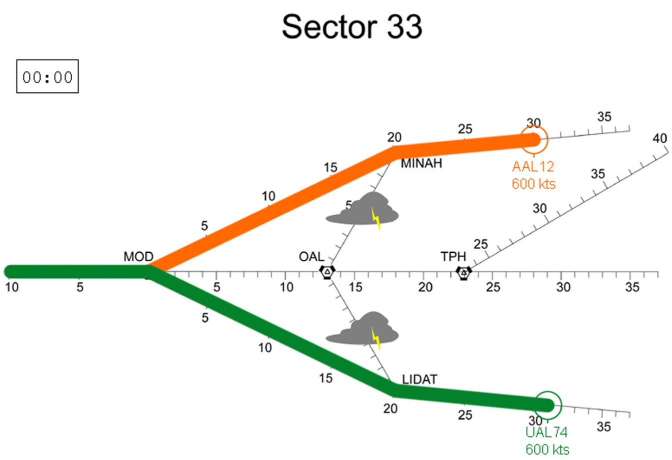


- **AAL12** - Slow down to 540 knots for 2 minutes to fall back 2 nautical miles. Then speed up to 600 knots.
- **Target Time -** 3 minutes and 48 seconds.

Problem 2-15

Solution

Starting Conditions:



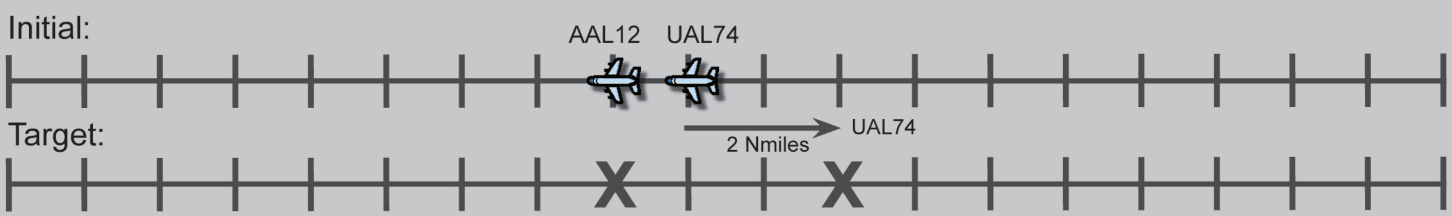
Plane	From	Through	To	Distance	Speed
AAL12	MINAH		MOD	30	600
UAL74	LIDAT		MOD	31	600

- Route from **LIDAT** to **OAL is closed**.
- Route from **MINAH** to **OAL is closed**.
- Ideal spacing at **MOD** is 3 nautical miles**.**

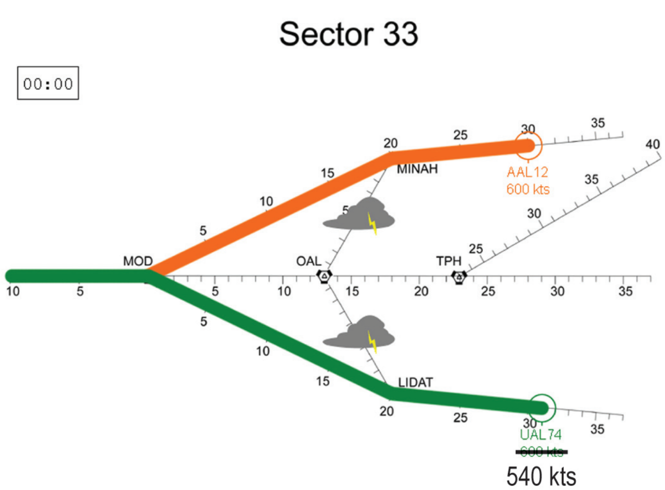
Analysis:

- **<u>Conflict:</u> UAL74** will arrive at MOD **1 nautical mile** behind **AAL12**.
- **Weather** prevents **UAL74** or **AAL12** from rerouting.
- **UAL74** needs to slow down to fall back 2 nautical miles.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	AAL12	30	1
2nd	UAL74	31	



Solution:

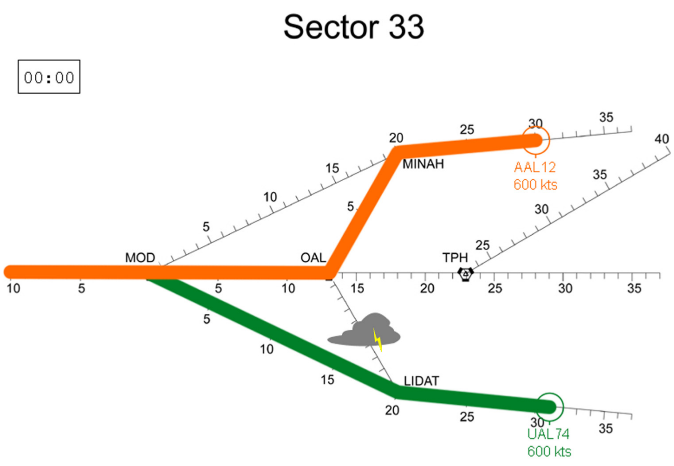


- **UAL74** - Slow down to 540 knots for 2 minutes to fall back 2 nautical miles. Then speed up to 600 knots.
- **Target Time** - 3 minutes and 18 seconds.

Problem 2-16

Solution

Starting Conditions:



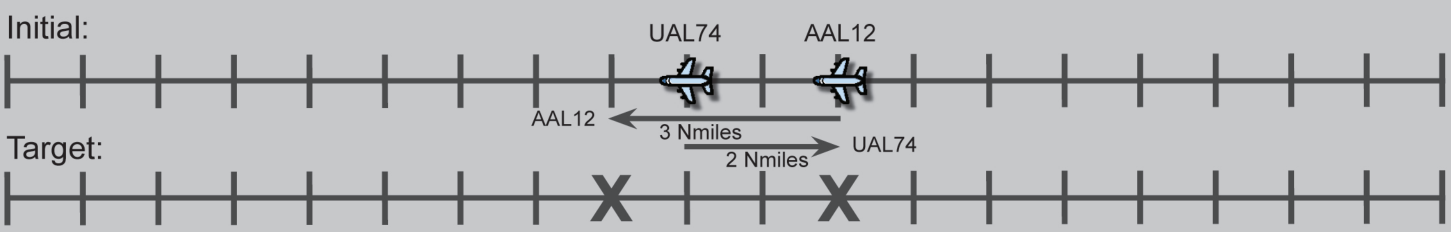
Plane	From	Through	To	Distance	Speed
AAL12	MINAH	OAL	MOD	33	600
UAL74	LIDAT		MOD	31	600

- Route from **LIDAT** to **OAL is closed**.
- Ideal spacing at **MOD** is 3 nautical miles**.**

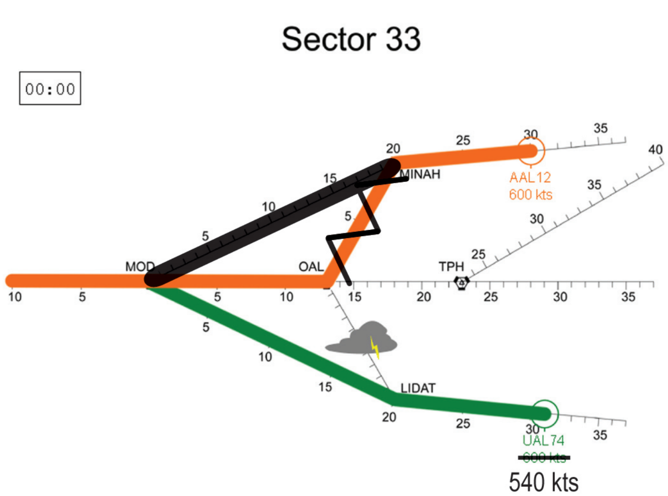
Analysis:

- **AAL12** will arrive at MOD **2 nautical miles** behind **UAL74**.
- **Weather** prevents UAL74 from rerouting
- **AAL12** can take the shortcut to shorten its travel distance by 3 nautical miles and move ahead of **UAL74** by 1 nautical mile. **UAL74** can slow down to fall back 2 nautical miles.

Projected Arrival	Plane	Distance Along Flight Plan	Initial Spacing
1st	UAL74	31	2
2nd	AAL12	33	



Solution:



- **AAL12** - Send directly to MOD to move forward 3 nautical miles.
- **UAL74** - Slow down to 540 knots for 2 minutes to fall back 2 nautical miles. Then speed up to 600 knots.

- **Target Time** - 3 minutes and 18 seconds.

LineUp With Math™

Math-Based Decisions in Air Traffic Control

Student Workbook E

Appendix II

- Resolving Air Traffic Conflicts by Changing Speed
  - 2 planes, each at the same starting speed
  - Simulator Problems 2-4, 2-5, 2-6, 2-7, 2-8

Workbook Answers

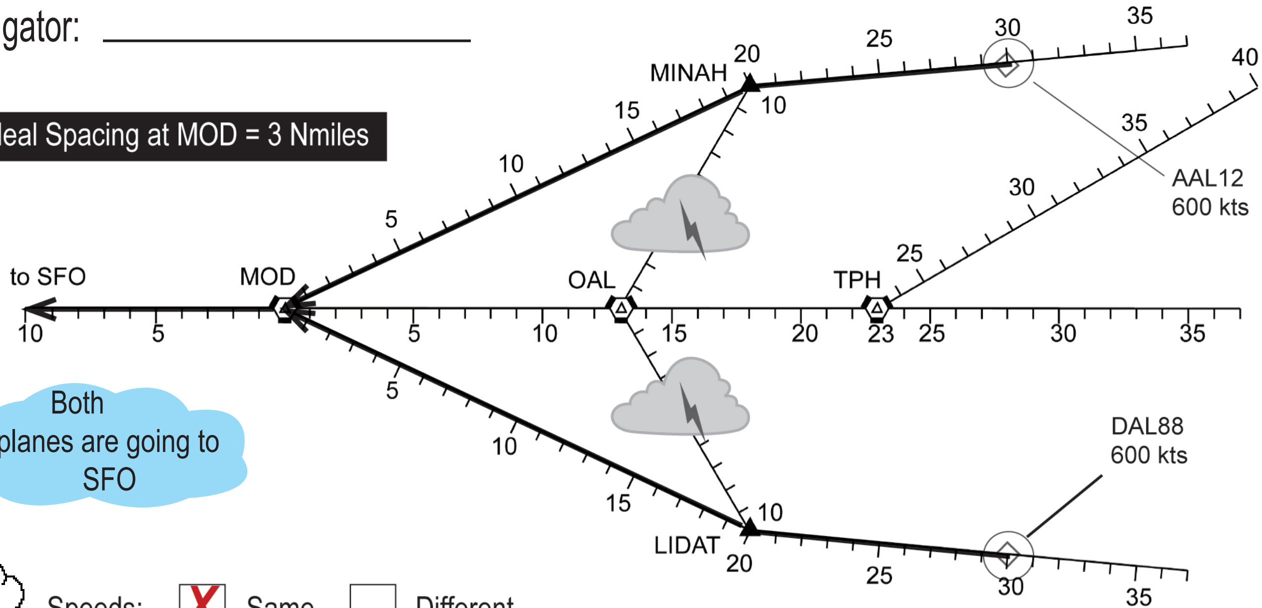
- Simulator at: www.atcsim.nasa.gov

Investigator: ______________________________

An Airspace Systems Program Product

Investigator: ______________________

Ideal Spacing at MOD = 3 Nmiles



Both planes are going to SFO

1. Speeds: [X] Same [ ] Different

2. Headstart = **0** Nmi = Separation at MOD

3. **Additional** Spacing Needed for Ideal Spacing (3 Nmiles) **3** Nmi

- To get the Ideal Spacing, you must change speed because the alternate routes are closed due to thunderstorms.

How Much Time Before You Need Ideal Spacing?

4. At 600 knots, how many minutes will it take the planes to reach MOD? **3** minutes

30 Nmi ÷ 10 Nmi/min = 3 mins

600 kts = 10 Nmi/Min

What Speed Change Will Solve the Problem?

- You can't speed up a plane because they are at the maximum speed of 600 knots.

5. Instead reduce the speed of one plane by 60 knots. Choose one plane to slow to 540 knots: ***Either plane***

Remember: * A 60 knot difference in speed causes a 1 nautical mile difference in distance each minute.

6. At 540 knots, how many nautical miles *less* will this plane travel *each minute*? **1** nautical miles *per minute*

7. In 3 minutes, how much *additional* spacing will you gain due to the speed reduction? **3** nautical miles

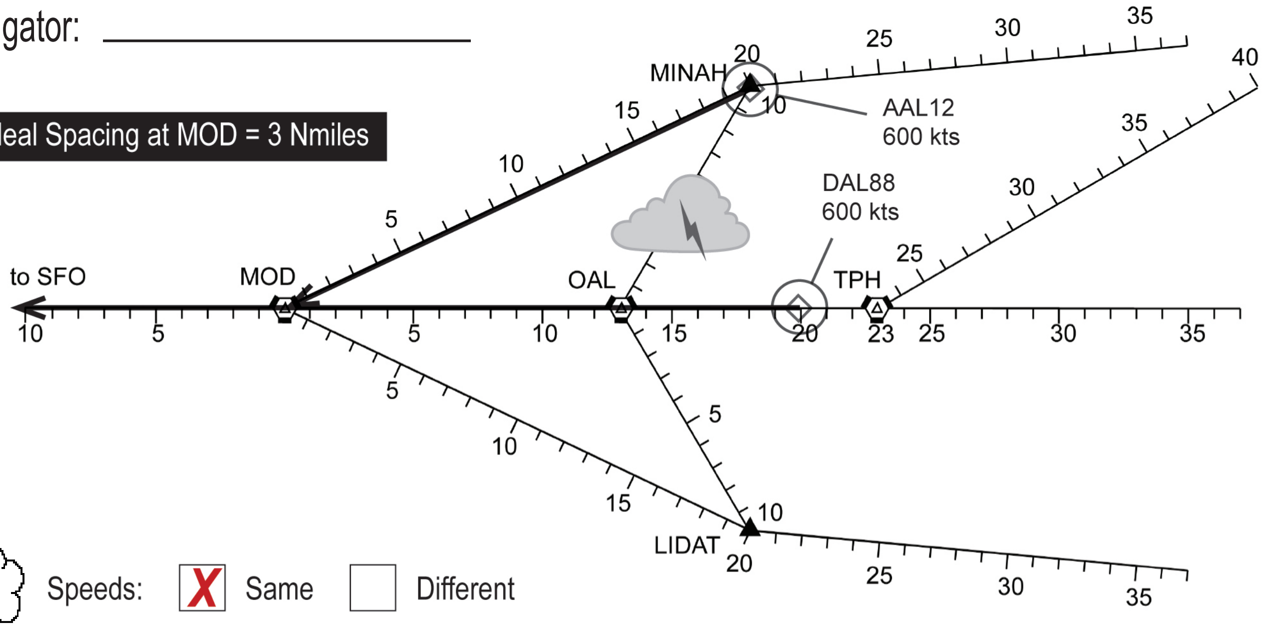
1 Nmi/min • 3 mins = 3 Nmi

8. Does the 60-knot speed drop give Ideal Spacing at MOD? [X] Yes [ ] No

End of Worksheet

Investigator: ____________________

Ideal Spacing at MOD = 3 Nmiles



1. Speeds: [X] Same [ ] Different

2. Spacing at MOD = **0** Nmi

3. Additional Spacing Needed for 3 Nmiles = **3** Nmi

- You must change speed to meet the Ideal Spacing.

600 kts = 10 Nmi/Min

4. At 600 knots, how many minutes will it take the planes to reach MOD? **2** minutes

20 Nmi ÷ 10 Nmi/min = 2 mins

Remember: * Controllers change speed in 60 knot steps.
* A 60 knot difference in speed causes a 1 nautical mile difference in distance each minute.

- First, slow AAL12 (or DAL88) by 60 knots, to 540 knots.

5. At MOD, how much spacing will you gain? **2** nautical miles ***1 Nmi/min • 2 min***

6. Did the 60-knot speed drop give you Ideal Spacing at or before MOD? [ ] Yes [X] No

- Try a greater speed drop. Slow AAL12 by 60 + 60 = 120 knots, to 480 knots.

7. Now how much spacing will you gain at MOD? **4** nautical miles ***2 Nmi/min • 2 min = 4 Nmi***

8. Did the 120-knot speed drop give you Ideal Spacing at MOD? [ ] Yes [X] No

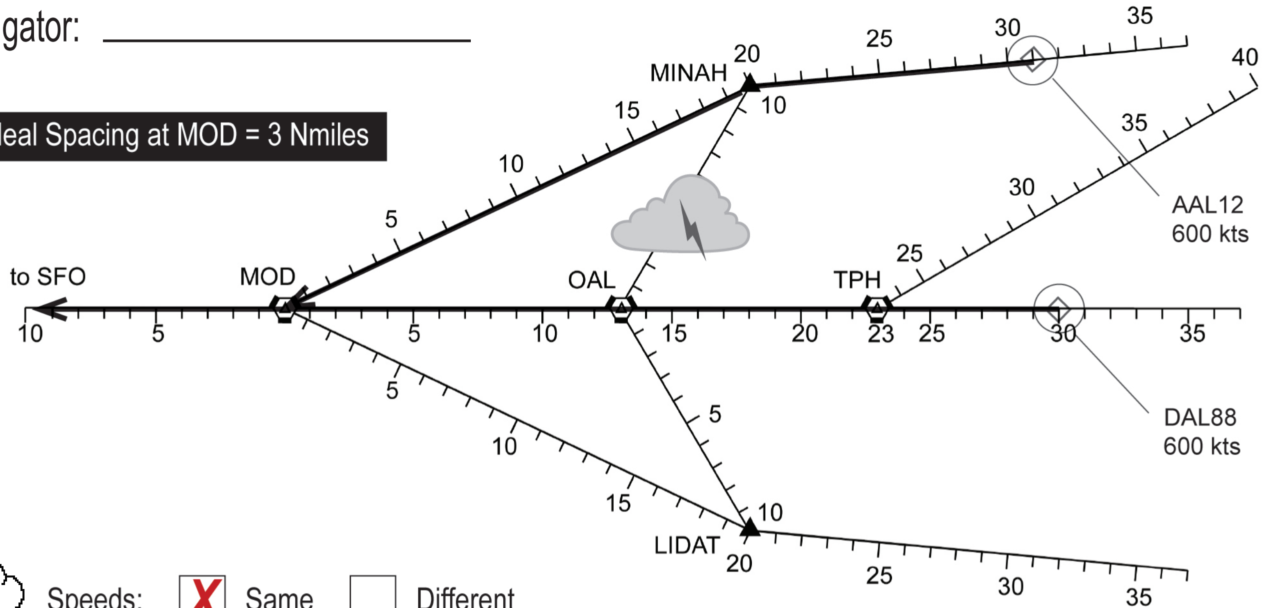
9. If No, what else could you do to get exactly Ideal Spacing at MOD?

Speed the plane back up when it has achieved 3 nautical mile separation.

End of Worksheet

Investigator: ____________________

Ideal Spacing at MOD = 3 Nmiles



1. Speeds: [X] Same [ ] Different

2. Spacing at MOD = **1** Nmi

   31 - 30 = 1 Nmi

3. Additional Spacing Needed for 3 Nmiles = **2** Nmi

4. At 600 knots, how many minutes will it take the **lead** plane to reach MOD? **3** minutes

   600 kts = 10 Nmi/Min

   30 Nmi ÷ 10 Nmi/min = 3 mins

- Controllers usually slow down the trailing plane (*not* the leading plane).

5. Which plane would a controller slow down to 540 knots? **AAL12** *(the furthest from MOD)*

- A 60 knot difference in speed causes a 1 nautical mile difference in distance each minute.

6. At this speed, how many nautical miles less will this plane travel each minute? **1** nautical miles *per minute*

7. At MOD, how much additional spacing will be gained due to the speed reduction? **3** nautical miles

   1 Nmi/min • 3 min = 3 Nmi

8. What is the new spacing at MOD? **4** nautical miles

   3 Nmi + 1 Nmi headstart = 4 Nmi

9. Is the spacing ideal? [ ] Yes [X] No

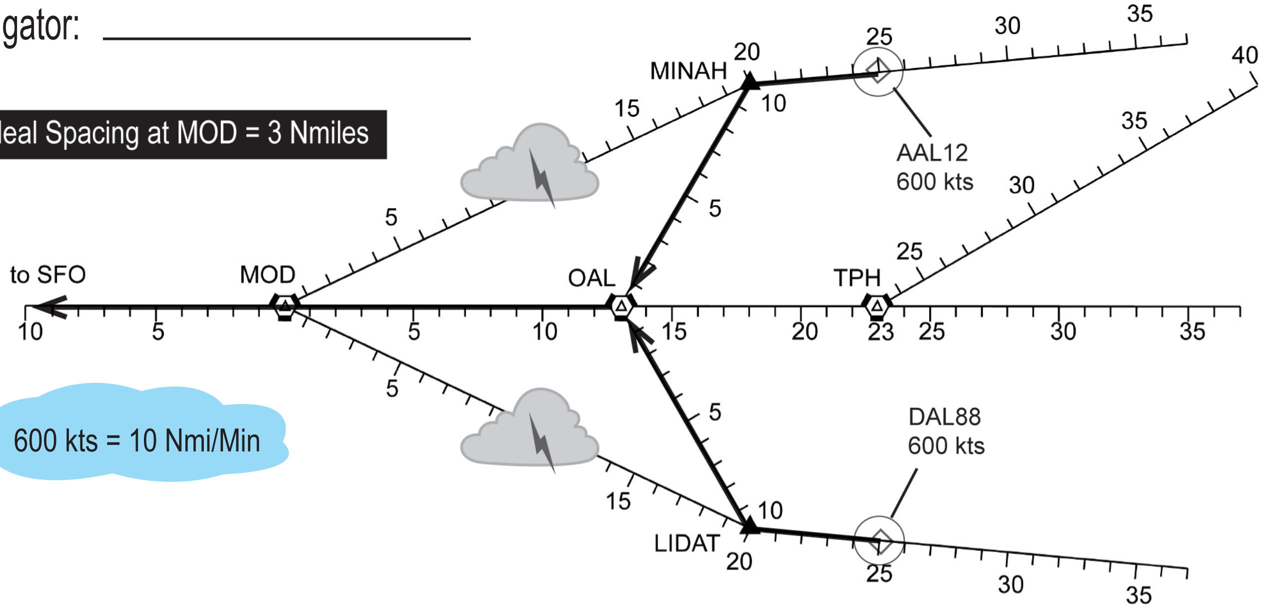
10. If no, after how many minutes will you speed the plane up to 600 knots to make the spacing ideal at MOD? **2** minutes

    With the 1 Nmi headstart, only need 2 Nmi extra.
    So speed up plane after 2 minutes.

End of Worksheet

Investigator: ____________________

Ideal Spacing at MOD = 3 Nmiles



600 kts = 10 Nmi/Min

Remember: * Controllers change speed in 60 knot steps.
* A 60 knot difference in speed will cause a 1 nautical mile difference in distance each minute.

- Analyze the problem at **OAL** (routes first meet). MUST meet or exceed **minimum** separation of 2 nautical miles.

1. Spacing at **OAL** = **0** Nmi

   15 - 15 = 0 Nmi

   Additional Spacing Needed for minimum separation of 2 Nmiles = **2** Nmi

- Let's solve the problem by slowing one plane. Let's slow that plane to 540 knots.

2. Which plane will you slow? **Either plane**

3. At **OAL**, how much additional spacing will be added due to the speed reduction? **1.5** nautical miles

   Faster plane will reach OAL in 1.5 mins. 1 Nmi/min • 1.5 mins = 1.5 Nmi

4. At 540 knots, will the planes have at least **minimum** separation of 2 nautical miles? [X] No [ ] Yes

   If No, what new speed will you use? **480** knots

5. At the new speed, what will the separation be at OAL? **3** nautical miles

   2 Nmi/min • 1.5 mins = 3 Nmi

6. At your final speed change, do you get at least **Minimum** Separation at OAL? [X] Yes [ ] No

7. If Yes, when will you speed the plane up to 600 knots to get Ideal Spacing at **MOD**?

   When faster plane arrives at OAL, speed up the slower plane to maintain 3 nautical miles.

End of Worksheet

EXTENSION

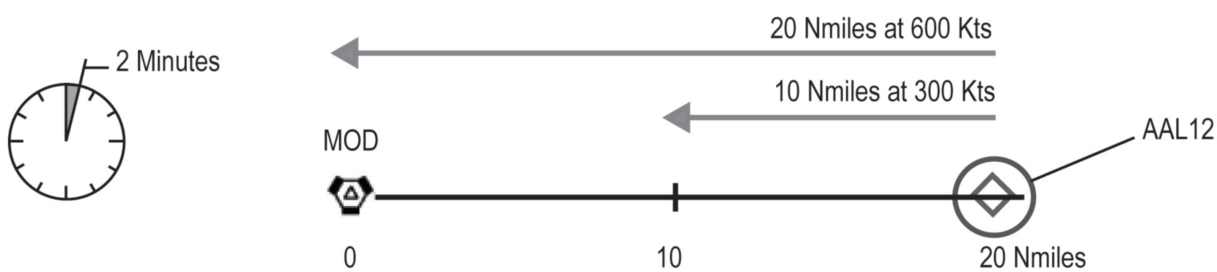
- Now we will use a new method, the Percent Rule, to solve speed change problems. Here's an example.

- At a speed of 600 knots, AAL12 travels 20 nautical miles to MOD in 2 minutes.

600 kts = 10 Nmi/Min

1 If we decrease the speed by 50% (that's ½ the speed), then the new speed is **300** knots

- At 300 knots (a 50% decrease in speed), AAL12 travels only 10 nautical miles (a 50% decrease) in 2 minutes.
- Here's a picture,



- So, in two minutes, we have:

Percent	Speed	Distance Traveled
100%	600 knots	20 nautical miles
50%	300 knots	10 nautical miles

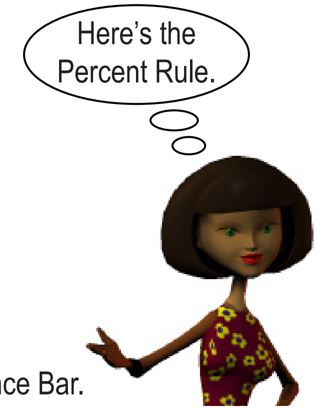
- The 50% decrease in speed gives a 50% decrease in distance traveled in the same time. This is an example of the Percent Rule:

> For a given amount of time, when you decrease a plane's speed by a given percent, the plane's distance traveled is decreased by the same percent.

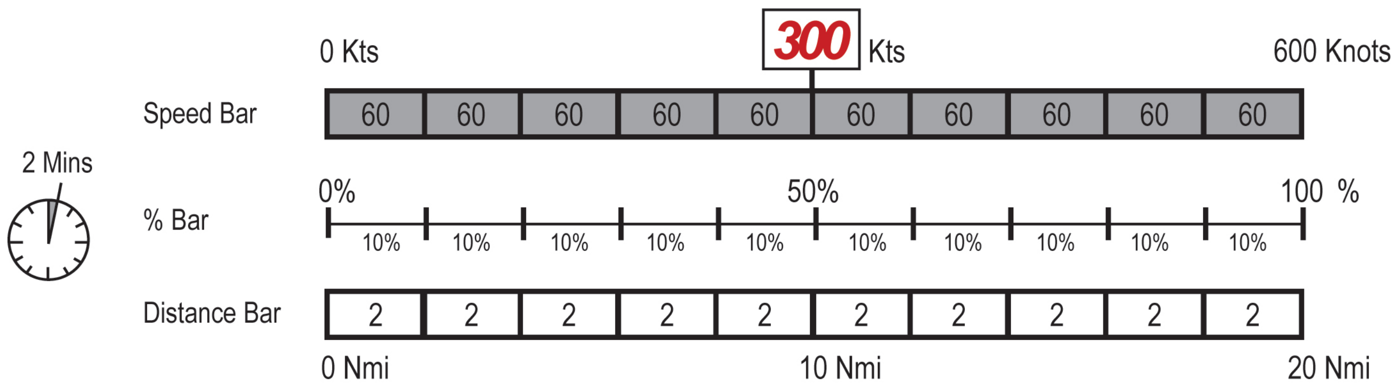
Continue to Next Page

The % Method (continued)

% decrease in speed = % decrease in distance traveled



- Now we will use the Percent Rule to get additional spacing at MOD.
- In the picture below, the plane's maximum speed, 600 knots, is shown in 10% intervals (60 knots each) on the Speed Bar.
- The plane is 20 nautical miles from MOD.
  The distance to MOD is shown in 10% intervals (2 nautical miles each) on the Distance Bar.



2 Above the Speed Bar, in the empty box, fill in the plane speed that is 50% of 600 knots.

- Use this picture and the Percent Rule to answer Questions 3 through 5.

3 If we decrease speed by 60 knots, what is the % decrease in speed? **10** %

4 Using the Percent Rule, what is the % decrease in distance traveled in two minutes? **10** %

5 How many **fewer** nautical miles will the plane travel in two minutes? **2** nautical miles

Continue to Next Page

The % Method (continued)

- Now suppose the plane is **30 nautical miles** from MOD, traveling at 600 knots.

	0 Kts				300 Kts					600 Knots
Speed Bar	60	60	60	60	60	60	60	60	60	60
% Bar (0% – 50% – 100 %)	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
Distance Bar	3	3	3	3	3	3	3	3	3	3
	0 Nmi				15 Nmi					30 Nmi

6 In the box below the Distance Bar, fill in the distance that is 50% of the **30 nautical miles** to MOD.

7 The distance to MOD is 30 nautical miles. For each 10% interval, fill each Distance Bar box with the number that is 10% of 30 nautical miles.

- Use this picture and the Percent Rule to answer Questions 8 through 12.

8 If we decrease speed by 120 knots, what is the percent decrease in speed? **20** %

9 Using the Percent Rule, what is the percent decrease in distance traveled in the same travel time? **20** %

10 Using this percent, how many **fewer** nautical miles will the plane travel? **6** nautical miles

$D = 20\% \text{ of } 30\text{ Nmi} = 0.2 \bullet 30\text{ Nmi} = 6\text{ Nmi}$

- Now the plane speed is **again** 600 knots.
The plane travels **30 nautical miles** to MOD in a certain amount of time.
But we don't need to know this time to answer this question.

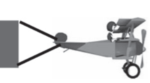
11 To travel 9 **fewer** nautical miles (in this same time) by what percent would you reduce the plane speed? **30** %

12 By how many knots would you reduce the plane speed? **180** knots

$3\text{ Nmi} + 3\text{ Nmi} + 3\text{ Nmi} \Rightarrow$

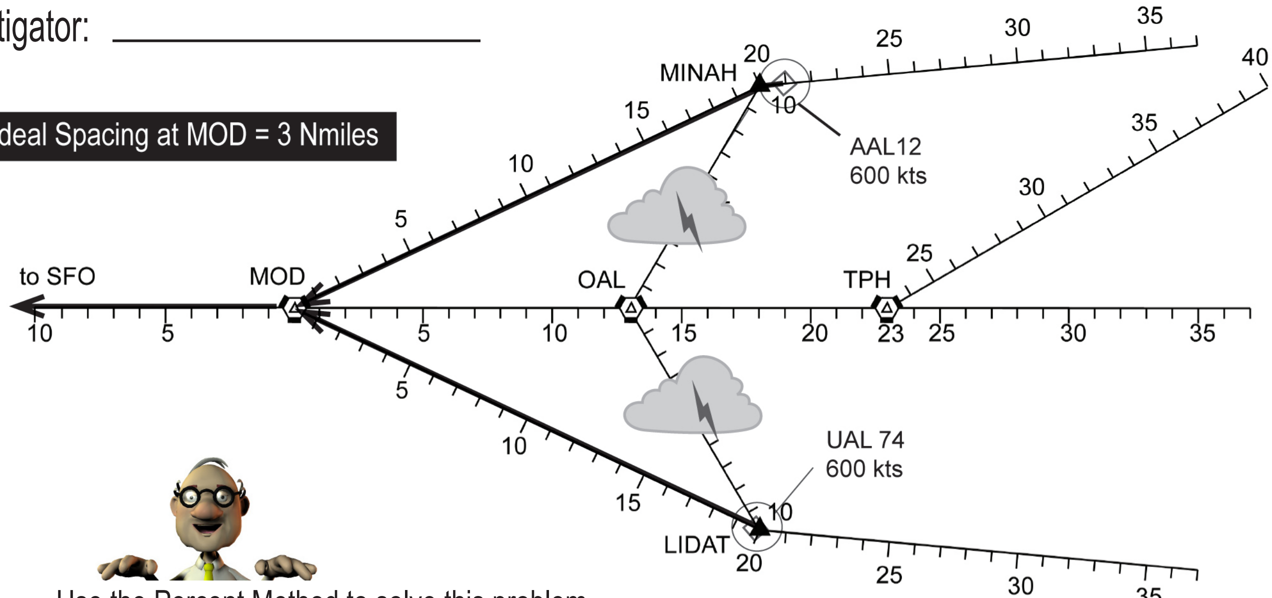
$60\text{ kts} + 60\text{ kts} + 60\text{ kts} = 180\text{ kts}$

End of Worksheet



Investigator: ____________________

Ideal Spacing at MOD = 3 Nmiles



- Use the Percent Method to solve this problem.

1. Lead plane = **UAL74** Spacing at MOD = **1** Nmi Additional Spacing Needed for 3 Nmiles = **2** Nmi

21 - 20 = 1 Nmi

- To achieve Ideal Spacing at MOD, decrease the speed of the trailing plane.

2. How many nautical miles does the *lead* plane travel to MOD? **20** nautical miles

3. When the lead plane reaches MOD, the *trailing* plane has traveled [X] the same [ ] a different distance.

- To get the additional spacing when the lead plane reaches MOD, decrease the trailing plane's 20-nautical-mile travel distance by **2** nautical miles.

4. What is the percent decrease in travel distance for the trailing plane?

$$\% \text{ Decrease} = \frac{\text{Additional Spacing Needed}}{\text{Distance Traveled}} = \frac{2 \text{ Nmiles}}{20 \text{ Nmiles}} = \frac{1}{10} = \mathbf{10}\ \%$$

5. For the trailing plane: to decrease its travel distance by 10%, decrease its speed by **10** %.

6. If you decrease the trailing plane's speed by 10%, what is its new speed? **540** knots

7. What is the new spacing at MOD? **3** nautical miles

600 - (10% of 600) =
600 - 60 = 540 kts

End of Worksheet

Appendix III

Derivation of Percent Method:

A given percent reduction in plane speed yields the same percent reduction in distance traveled in the original amount of time.

To derive this percent relationship between reduced speed and reduced distance, we use the formula

$$\text{distance} = \text{rate} \bullet \text{time}.$$

Let d_1, r_1, and t be the original distance, speed, and time. Then

$$d_1 = r_1 \bullet t$$

We solve this equation for r_1 to obtain an expression for the original speed.

$$r_1 = d_1 / t$$

Let d_2 and r_2 be the reduced distance and speed, respectively. Since we are concerned with the distance covered in the *original* amount of time, t, we again use t to represent time. We have

$$d_2 = r_2 \bullet t$$

That is, $$r_2 = d_2 / t$$

Recall, % decrease in speed = 100 • (original speed - reduced speed) ÷ original speed

So,

$$\begin{aligned}\%\text{ decrease in speed} &= 100 \bullet (r_1 - r_2) \div r_1\\ &= 100 \bullet (d_1 / t - d_2 / t) \div (d_1 / t)\\ &= 100 \bullet (d_1 - d_2) / t \div (d_1 / t)\\ &= 100 \bullet (d_1 - d_2) / t \bullet (t / d_1)\\ &= 100 \bullet (d_1 - d_2) \div d_1\end{aligned}$$

Thus,

$$\%\text{ decrease in speed} = 100 \bullet (d_1 - d_2) \div d_1$$

Similarly,

% decrease in distance = 100 • (original distance - reduced distance) ÷ original distance

So,

$$\%\text{ decrease in distance} = 100 \bullet (d_1 - d_2) \div d_1$$

Thus we see the % decrease in speed is equal to the % decrease in distance.